Price Setting on a Network*

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Abstract

I study price setting on a network of interconnected firms. Some or all these firms may have market power. The key distortion reducing both total profits and social welfare is multiple-marginalization, which is magnified by strategic interactions. Individual profits are proportional to influentiality, a new measure of network centrality defined by the equilibrium characterization. The results emphasize the importance of the network structure when considering policy questions such as mergers or trade policies.

JEL: C72, L14, D43

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1 Introduction

Most products are produced and sold by supply chain networks consisting of interconnected producers, intermediaries, and retailers. These firms maximize their profits, often

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wielding significant market power. For instance, in the book publishing industry, a publisher sources content, outsources printing, and employs distributors to reach retail chains. Many of these players possess market power and earn positive profits. [1]

This paper aims to answer the question: How do firms in a network set prices for their products when they have market power? In my model, I assume that firms not only control the pricing of their own products, but also influence the pricing decisions of other firms within the network. However, characterizing the equilibrium presents two challenges. Firstly, the demand function can be non-linear, making it difficult to solve for best-response functions analytically many networks. Secondly, the decisions made by firms within the network are interconnected. This means that the problem is dynamic, but cannot be solved sequentially. To address these issues, I characterize best-responses by their inverses and aggregate all necessary conditions into one necessary condition for the equilibrium. I then demonstrate that this condition has a unique solution and that the resulting behavior is indeed an equilibrium.

The main result of the paper is a characterization theorem. Under certain regularity conditions, there exists a unique equilibrium. I provide equilibrium characterization and show how to compute it.The equilibrium condition has a natural interpretation. It equalizes the difference between the equilibrium price of the final good and the total marginal cost with a weighted sum of *influences* in all levels. At the most basic level, a firm's profit is directly affected by its own price increase. At the next level, a firm's price increase can alter the behavior of directly connected firms. Furthermore, each price change can have a ripple effect, influencing firms that are indirectly connected. These influences are weighted by endogenous factors determined by the demand function's shape and the equilibrium behavior.

How do social welfare and total profits depend on network structure? In this model of interconnected firms with market power, the primary distortion that diminishes both profits and welfare is multiple marginalization. The main insight from the analysis is that network structure plays a pivotal role in determining the magnitude of the marginalization issue. I show that strategic interactions within a network magnify the marginalization problem. For instance, while a merger might seem to enhance efficiency based on conventional wisdom, if it leads to increased control by the merged firms, it could potentially negate the anticipated efficiency gains. Likewise, trade restrictions and tariffs, often crafted to modify supply chains, should be assessed with an understanding of their effects on the business network structure.

The rest of the paper is structured as follows. The next section discusses the related literature. Section 3 introduces the model, illustrates it with some examples of networks, and discusses the regularity assumptions. Section 4 provides the main characterization result and describes the main components of the proof—in particular, how the characterization overcomes two main complications that arise from non-linearities in demand and interconnected decisions on the network. Section 5 interprets the characterization by comparing it with some known benchmarks and then discusses the key distortion—multiple-marginalization. Section 6 studies which firms are more influential and discusses

[1] In a typical \$26 book, retailers take about 50%, 13% covers printing and transport, and authors receive around 15%, indicating significant markups. Source: New York Times article "Math of Publishing Meets the E-Book" by Motoko Rich (Feb. 28, 2010), `https://www.nytimes.com/2010/03/01/business/media/01ebooks.html`.

the relationship between the implied influentiality measure with standard network centrality measures. Section 7 describes how to apply the characterization result to compute the equilibrium and provides further results for some of the most common demand functions (including linear, power, and logit). Section 8 concludes and discusses policy implications for mergers and trade. All proofs are in appendix A.

2 Related Literature

Industrial organization. The paper contributes to the literature on vertical integration. Spengler (1950) was the first to describe the double-marginalization problem and after this the literature has extensively studied the benefits and costs of vertical control, including Mathewson and Winter (1984), Grossman and Hart (1986), Rey and Tirole (1986), Salinger (1988), Salinger (1989), Riordan (1998), Ordover et al. (1990), Farrell and Shapiro (1990), Bolton and Whinston (1993), Kuhn and Vives (1999), Nocke and White (2007), and Buehler and Gärtner (2013). Empirical work shows that production has a network structure (Atalay et al., 2011), and mergers or removal of vertical restraints may sometimes hurt consumers (Gayle, 2013; Crawford et al., 2018; Luco and Marshall, 2020). The theoretical literature analyzes many forms of competition and contract structures, but very little is known about networks with firms with market power at more than two levels (upstream-downstream).[2] In this paper, I focus on a simple contract structure (posted prices) and allow only relatively simple competition rules (either price-takers or monopolists), but extend the analysis to general network structures. I demonstrate that both direct and indirect strategic interactions on networks magnify multiple-marginalization distortions.[3]

Network games. The paper contributes to the literature on network games, where players take actions on a fixed network and the payoffs depend both on their own and their neighbors' actions. According to a survey by Jackson and Zenou (2015), most works in this literature can be divided into two groups. First, a lot of progress has been made in games with quadratic payoffs (or more generally, payoffs that imply linear best-responses). A seminal paper is Ballester et al. (2006). It found that the equilibrium actions are proportional to Bonacich centrality. Bramoullé and Kranton (2007), Calvó-Armengol et al. (2009), Bramoullé et al. (2014), and Zhou and Chen (2015) study more general variations of this game and find that Bonacich centrality still determines the equilibrium behavior. Bloch and Quérou (2013) and Fainmesser and Galeotti (2016) study pricing of goods with network externalities with quadratic payoffs and find that optimal pricing leads to discounts that are proportional to Bonacich centrality. Bimpikis et al. (2019) study Cournot competition on a bipartite network, where the sellers Cournot-compete in

[2]Nava (2015) is an exception that studies Cournot competition when trades are restricted by the network structures and provides a characterization result. While the set-up is different from mine, it also identifies marginalization as a major source of inefficiency. However, the inefficiency disappears with a large number of firms, whereas my model does not have this feature.

[3]Supply chain management literature, originating with Forrester (1961), has similar effects like the bullwhip effect (Lee et al., 2004; Bhattacharya and Bandyopadhyay, 2011; Liu et al., 2007; Perakis and Roels, 2007), where chain interactions amplify distortions. Recent studies highlight chain fragility (Acemoglu and Tahbaz-Salehi, 2020; Elliott et al., 2022).

markets which they have access to. They show that when the demands are linear and costs quadratic, the equilibrium behavior is proportional to Bonacich centrality.

The second branch of network games studies games with non-quadratic payoffs and is generally able to analyze only qualitative properties of the equilibria rather than provide a full characterization.[4] A seminal paper is Galeotti et al. (2010). Compared to these works, in this paper, I provide a characterization result for a game on a network with a relatively general payoff structure. The characterization defines a natural new measure of influentiality and the firms' choices and payoffs are proportional to this measure. In special cases when the best-response functions are linear (such as linear demand), this measure is proportional to Bonacich centrality. But as the weights are endogenously defined by the demand function and equilibrium behavior, for all other demand functions the measure of influentiality differs from Bonacich centrality. Indeed, I provide examples of special cases where it can be equivalent to degree centrality or even independent of the network structure.[5][6]

Sequential and aggregative games. Methodologically, the paper builds on recent advances in sequential and aggregative games. In a special case, when firms are making independent decisions, the model is an aggregative game. Aggregative games were first proposed by Selten (1970) and there has been recent progress in aggregative games literature by Jensen (2010); Martimort and Stole (2012); and Acemoglu and Jensen (2013) that has been used to shed new light on questions in industrial organization by Nocke and Schutz (2018).One classical aggregative game is a contest and this paper builds on recent work on sequential contests by Hinnosaar (2023) extending the methodology to networks and asymmetric costs.[7]

3 Model

3.1 Setup

The model is static and studies the supply of a single final good. The final good has a demand function $D(P)$, where P is its price. The production and supply process requires

[4]An exception is Choi et al. (2017), which studies price competition on networks, where consumers choose the cheapest paths from source to destination and intermediaries set prices, thus making the game a generalization of Bertrand competition. In settings where the players interact on a network randomly, the analysis is more tractable, for example Manea (2011), Manea (2018), and Condorelli et al. (2017) who study bargaining on networks.

[5]There is an earlier literature on supply chain networks in economics, started by Hatfield and Milgrom (2005); Ostrovsky (2008). This literature focuses on market design and matching, i.e., network formation, whereas my paper belongs to network games literature analyzing behavior on a fixed network.

[6]Networks also play an important role in international trade and macroeconomics. Trade exhibits a network structure (Chaney, 2014), with early works integrating vertical restraints and intermediaries' roles (Spencer and Jones, 1991; Antràs and Costinot, 2011). Recent literature focuses on network formation in production (Oberfield, 2018; Liu, 2019).

[7]There are other papers belonging to the intersection of contests and networks literature, including Franke and Öztürk (2015) and Matros and Rietzke (2018) who study contests on networks and Goyal et al. (2019) who study contagion on networks.

$m+n$ inputs. I normalize the units of inputs so that one unit of each input is required to produce one unit of output.

Input i is produced by firm i, that has a constant marginal cost c_i and a price p_i for its product.The price p_i is firm i's per-unit revenue net of payments to other firms in the model. Due to normalization, the quantity of firm i's product (i.e., quantity of input i) is equal to $D(P)$. Therefore firm i gets profit $\pi_i(\boldsymbol{p}) = (p_i - c_i)D(P)$, where $\boldsymbol{p} = (p_1, \dots, p_{m+n})$ and the price of the final good is the sum of all net prices, $P = \sum_{i=1}^{m+n} p_i$.

I assume that m inputs $1, \dots, m$ are produced by *monopolists* (referred to as $1, \dots, m$), who set their prices strategically, i.e., maximizing profits, anticipating the impact on sales of the final good. The remaining n inputs $m+1, \dots, m+n$ are produced by *price-takers* (non-strategic firms), who treat their prices as fixed. A price-taker i may operate in a competitive sector or compete as a Bertrand competitor, in which case its price is equal to the marginal cost of the second cheapest firm in this sector. The firm could also operate in a regulated industry and its price is set by a regulator.

To complete the description of the model, I need to specify how the price p_i of firm i affects the behavior of other monopolists, which I do by introducing the network of influences. Formally, a network of influences consists of all m monopolists as nodes and edges that define influences. The edges are described as an $m \times m$ adjacency matrix $\boldsymbol{A}$, where an element $a_{ij} = 1$ indicates that firm i *influences* firm j. That is, when firm j chooses price p_j, then it takes price p_i as given and responds optimally to it. Of course, firm i knows this and when choosing p_i, it knows that j will respond optimally. Finally, if i and j are not directly linked, i.e., $a_{ij} = a_{ji} = 0$, then neither responds to deviations by the other firm. They expect the other firm to behave according to its equilibrium strategy. For convenience, I assume that the diagonal elements $a_{ii} = 0$. I will discuss a few examples of the network of influences in the next subsection.

Let me make four remarks about the model here. First, the network of influences is a *reduced-form* way to capture sequential interactions. I will discuss more examples in the next subsection, but one natural way to interpret it is through the lens of commitment power: some monopolists may have more commitment power than others in their pricing decisions. An alternative interpretation is bargaining power: they can make take-it-or-leave-it offers. In this paper, I take the network as a fixed primitive of the model and do not explicitly model its microfoundations.

Second, the price-takers are non-strategic players, so without loss of generality I replace them by a single parameter $c_0 = \sum_{i=m+1}^{m+n} p_i$. Parameter c_0 can be interpreted as a cost for the supply chain. I denote the total per-unit cost to the supply chain by $C = c_0 + \sum_{i=1}^{m} c_i$.

Third, the analysis does not require that each firm on the supply chain is either always a price-taker or always a monopolist. The assumption that I use in the characterization is that monopolists behave according to their local optimality condition, whereas price-takers take their prices locally as given. A firm could be a monopolist in one situation and a price-taker when the model parameters change.

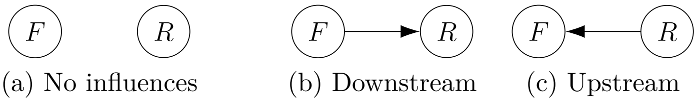
Fourth, the network of influences makes the game sequential. If $a_{ij} = 1$ then firm i sets its price p_i before firm j. Firm j then observes p_i and may respond optimally. Of course, firm i knows this and therefore can anticipate the response of firm j. I am looking for pure-strategy perfect Bayesian equilibria, where players take some of the choices of other players as given and maximize their profits, anticipating the impact on other players' choices and the final good demand.[8]

3.2 Examples of Network of Influences

The network of influences I introduce in this paper is related to but not the same as the supply-chain network. A typical supply-chain network specifies the flows of goods and services (material flows), as well as the flows of money and information. The specifics of these flows are neither necessary nor sufficient to characterize pricing decisions.[9] For pricing decisions, the model needs to specify what is known to each monopolist at the moment it makes a pricing decision and how it expects this decision to influence the choices of other firms. In other words, the model needs to specify the observability of prices and the commitment power of firms. As described above, I model this by assuming that there is a commonly known network, such that whenever there is an edge from i to j, firm j observes p_i and therefore takes it into account in its optimization problem.

Consider first a very simple case with just two firms, F (final goods producer) and R (retailer). Then there are three possible networks, illustrated by figure 1. First, figure 1a where firms set their prices p_F and p_R independently, and the final good is sold at $P = p_F + p_R$. This could be a reasonable assumption, for example, if both are large firms that interact with many similar firms. Then the final goods producer F does not best-respond to a particular retailer R but to the equilibrium price p_R^* of a representative retailer. Similarly, the retailer does not best-respond to deviations by particular producer F, but to equilibrium price p_F^* of a representative producer. Another example where it is natural to make this assumption is when two firms are separately selling perfectly complementary products to final consumers.

Similarly, there could be many reasons for strategic influences. For example, a downstream influence from producer F to retailer R (figure 1b) may arise with a large producer and small retailer, where the representative retailer reacts optimally to pricing by F. The large producer knows that retailers respond to its pricing and therefore takes into account how a representative retailer best-responds. Of course, the influence could go in the opposite direction (as in figure 1c) for the same reason—a large retailer R knows that a small producer F will best-respond to its price changes. In this paper, I take these influences as given and simply assume that some firms have more commitment power than others for exogenous reasons.



(a) No influences (b) Downstream (c) Upstream

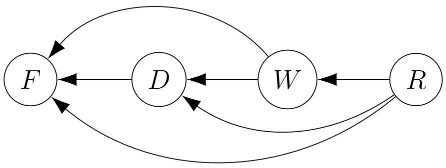
Figure 1: Example: Three possible two-player networks.

Let me illustrate the network of influences with three more examples. Figure 2 depicts an example of a retail chain with downstream-to-upstream influences. In this example,

[8] Although I am not excluding the possibility of mixed-strategy equilibria, I show that there always exists a unique pure-strategy equilibrium, so it is natural to focus on it.

[9] Indeed, my model can be alternatively interpreted as a system of perfectly complementary products, where producers make their pricing decisions over time (Matutes and Regibeau, 1992).

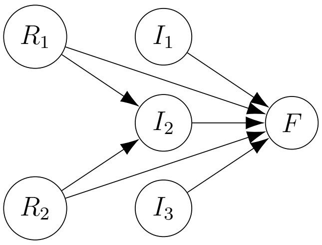
there is a strong retailer R, who can commit to adding a markup p_R on top of the wholesale price P_W, so that the price of the final good will be $P = P_W + p_R$. The wholesaler W takes p_R as given and commits to its markup p_W, so that when the distributor's price is P_D, then wholesale price is $P_W = P_D + p_W$ and therefore final good price $P = P_D + p_W + p_R$. Then distributor D sets its markup p_D taking markups p_W and p_R as given. Finally, the final good producer F sets a price p_F, taking into account that final consumer will pay $P = p_F + p_D + p_W + p_R$.



$$\boldsymbol{A} = \begin{array}{cc} & \begin{array}{cccc} F & D & W & R \end{array} \\ \begin{array}{c} F \\ D \\ W \\ R \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Figure 2: Example: Retail chain with downstream-to-upstream influences.

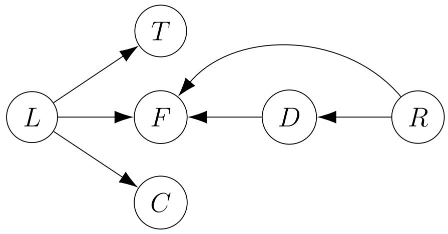
Influences can also go in the opposite direction. Figure 3 gives an example of a production chain. In this example, there is a small producer F who produces the final good and uses three inputs, produced by intermediate good producers I_1, I_2, and I_3. Firm F takes the prices of its inputs P_{I_1}, P_{I_2}, and P_{I_3} as given and chooses the price for the final good, $P_F = P$. Intermediate good producer I_2 uses two raw materials as inputs, produced respectively by R_1 and R_2. In this example, firm I_2 when choosing P_{I_2} takes P_{R_1} and P_{R_2} as given. Importantly, as firms I_1 and I_3 do not use these inputs, they do not know the realized prices offered by R_1 and R_2, but they can make equilibrium conjectures. Therefore, while firms I_1 and I_3 take into account the equilibrium prices $P^*_{R_i}$ in their optimization problem, they cannot respond to potential deviations in P_{R_i}, whereas firms I_2 and F can and do respond to these deviations. It is convenient to redefine prices as net prices, net of transfers to the other firms, i.e., $p_{R_1} = P_{R_1}, p_{R_2} = P_{R_2}, p_{I_1} = P_{I_1}, p_{I_2} = P_{I_2} - p_{R_1} - p_{R_2}$, $p_{I_3} = P_{I_3}$, and $p_F = P - p_{I_1} - p_{I_2} - p_{I_3} - p_{R_1} - p_{R_2}$, so that $\sum_i p_i = P$.



$$\boldsymbol{A} = \begin{array}{cc} & \begin{array}{cccccc} R_1 & R_2 & I_1 & I_2 & I_3 & F \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ I_1 \\ I_2 \\ I_3 \\ F \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Figure 3: Example: Production chain with upstream-to-downstream influences.

There is no reason to assume that the flows of influence are all going in the same direction or that the network is a tree. Figure 4 gives another example, where the same raw material L (labor) is used by three firms, T (transport), F (final goods producer), and C (communication). These three firms set their prices independently, but F additionally takes the markups of the D (distributor) and R (retailer) as given.



$$
\boldsymbol{A} = \begin{array}{c} \\ L \\ T \\ F \\ C \\ D \\ R \end{array}
\begin{array}{c}
\begin{array}{cccccc} L & T & F & C & D & R \end{array} \\
\begin{bmatrix}
0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

Figure 4: Example: a network with a small producer and a common raw-material producer

3.3 Regularity Assumptions

I make three technical assumptions that are sufficient for the existence and uniqueness of the equilibrium. The first assumption specifies the class of networks.

Assumption 1. *Network $\boldsymbol{A}$ is acyclic and transitive.*[10]

It is natural to assume that the network is acyclic. If firm j takes p_i as given, then it simply cannot be that firm i takes its decision p_j as given. A similar argument applies to cycles of more than two players. Note the assumption allows firms i and j to make independent choices when $a_{ij} = a_{ji} = 0$ and the network does not have to be connected.[11]

The transitivity requires that if i influences j and j influences k, then i also influences k directly, i.e., k takes both p_j and p_i as given. In the examples above this was a natural assumption. Relaxing transitivity assumption would add the possibility of signaling to the game. For example, suppose that in the network described by figure 4 there is no edge from R to F. Then firm F knows that p_R will be added to the price, but does not know the value. However, since F knows p_D and D knows p_R, the price p_D may reveal some information about p_R. Transitivity assumption excludes such signaling possibilities and thus simplifies the analysis significantly.[12]

The second regularity assumption puts standard restrictions on the demand function. The demand function $D(P)$ is a smooth and strictly decreasing function. It either has a finite saturation point $\overline{P}$ at which the demand is zero or converges to zero fast enough so that profit maximization problem is well-defined.

Assumption 2. *Demand function $D : [0, \overline{P}) \to \mathbb{R}_+$ is continuously differentiable and strictly decreasing in $[0, \overline{P})$ where $\overline{P} \in \mathbb{R}_+ \cup \{\infty\}$. Moreover, it satisfies limit condition $\lim_{P \to \overline{P}} PD(P) = 0$.*

The third and final regularity assumption ensures that the demand function $D(P)$ is well-behaved so that the optimum of each firm can be found using the first-order condition. It is common in the literature to make a regularity assumption that D is twice differentiable and profits single-peaked. In particular, in theoretical works the demand is

[10] Acyclicity: $\nexists i_1, \ldots, i_k$ such that $a_{i_1 i_2} = \cdots = a_{i_{k-1} i_k} = a_{i_k i_1} = 1$. Equivalently, $\boldsymbol{A}^m = \mathbf{0}$, where m is the number of monopolists. Transitivity: if $a_{ij} = a_{jk} = 1$, then $a_{ik} = 1$. Equivalently, $\boldsymbol{A} \geq \boldsymbol{A}^2$.

[11] Acyclicity is also helpful in terms of tractability. Recent papers by Galeotti et al. (2021); Pellegrino (2023) have related models without acyclicity, but they do not allow for sequential decisions.

[12] Compared to typical signaling models, the private information here is about the choices of other players (deviations in particular) rather than some underlying uncertainty.

often assumed to be linear for tractability. However, in empirical literature logit demand is more common. Here I make an assumption about the demand function that would be analogous to the standard regularity assumption and covers both linear and logit demand functions.

Let the depth of the network $d(\boldsymbol{A})$ be the length of the longest path in $\boldsymbol{A}$.[13] For example, in figure 4 depth $d(\boldsymbol{A}) = 3$ (from the path $R \to D \to F$). Moreover, let me define a function

$$g(P) = -\frac{D(P)}{D'(P)}, \tag{1}$$

which is a convenient alternative way to represent the demand function. Note that $g(P) = \frac{P}{\varepsilon(P)}$, where $\varepsilon(P) = -\frac{dD(P)}{dP}\frac{P}{D(P)}$ is the demand elasticity.[14] Then I make the following assumption about the shape of the demand function.

Assumption 3. *$g(P)$ is strictly decreasing and $d(\boldsymbol{A})$-times monotone in $P \in (0, \overline{P})$, i.e., for all $k = 1, \ldots, d(\boldsymbol{A})$, derivative $\frac{d^k g(P)}{dP^k}$ exists and $(-1)^k \frac{d^k g(P)}{dP^k} \geq 0$ for all $P \in (0, \overline{P})$.*

To interpret the condition, let us look at the standard monopoly pricing problem $\max_P \pi(P) = \max_P (P - C)D(P)$. Then the first-order necessary condition for optimality of P^* is

$$\pi'(P^*) = D(P^*) + (P^* - C)D'(P^*) = 0 \quad \Longleftrightarrow \quad P^* - C = g(P^*), \tag{2}$$

which illustrates the convenience of the $g(P)$ notation. Moreover, a sufficient condition for optimality is $\pi''(P^*) < 0$ or equivalently $2[D'(P^*)]^2 > D(P^*)D''(P^*)$. Note that a sufficient condition for this is $[D'(P^*)]^2 > D(P^*)D''(P^*)$, which is equivalent to $g'(P^*) < 0$. Therefore in the standard monopoly problem, monotonicity of $g(P)$ guarantees that monopoly profit has a unique maximum that can be found using the first-order approach. For general networks, the condition is stronger, as it also guarantees that best-responses and best-responses to best-responses are well-behaved so that the first-order approach is valid.

As illustrated by the monopoly example, the condition is sufficient and not necessary, but it is easy to check and it is satisfied for many applications. The following proposition provides a formal statement by showing that with many typical functional form assumptions on $D(P)$, the function $g(P)$ is completely monotone, i.e., d-times monotone for arbitrarily large $d \in \mathbb{N}$. Therefore assumption 3 is satisfied with all networks.

Proposition 1 (Many demand functions imply completely monotone $g(P)$)**.** *Each of the following demand functions implies d-times monotone $g(P)$ for any $d \in \mathbb{N}$:*

1. *Linear demand $D(P) = a - bP$ with $a, b > 0 \Rightarrow g(P) = \overline{P} - P$, where $\overline{P} = \frac{a}{b} > 0$.*

2. *Power demand $D(P) = d\sqrt[\beta]{a - bP}$ with $d, \beta, a, b > 0 \Rightarrow g(P) = \beta(\overline{P} - P)$.*

3. *Logit demand $D(P) = d\frac{e^{-\alpha P}}{1+e^{-\alpha P}}$ with $d, \alpha > 0 \Rightarrow g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right]$.*

4. *Exponential demand $D(P) = a - be^{\alpha P}$ with $a > b > 0, \alpha > 0 \Rightarrow g(P) = \frac{1}{\alpha}\left[\overline{P}e^{-\alpha P} - 1\right]$.*

[13] Formally, $d(\boldsymbol{A})$ is smallest d is such that $\boldsymbol{A}^d = \boldsymbol{0}$.

[14] Technically, $g(P)$ is the reciprocal of the demand semi-elasticity.

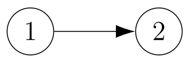
Note that for all four functions assumption 2 is clearly also satisfied. Linear and power demand functions have saturation point $\overline{P}$, logit demand satisfies $\lim_{P\to\infty} PD(P) = d\lim_{P\to\infty}\frac{1}{\alpha e^{\alpha P}} = 0$, and exponential demand has saturation point $\overline{P} = \frac{1}{\alpha}\log\frac{a}{b}$.[15]

4 Characterization

In this section, I first discuss two examples illustrating the complications arising from the non-linearities and the network structure. I use these examples to illustrate the techniques that I use for equilibrium characterization. The main result of the paper is the characterization theorem that formalizes the approach.

4.1 Example: Non-linear Demand

The first example illustrates the complexities that arise from working with a non-linear demand function when the decisions are sequential. Let us consider logit demand $D(P) = \frac{e^{-P}}{1+e^{-P}}$, costless production, and two monopolists, who choose their prices sequentially, as indicated in figure 5. That is, first firm 1 chooses price p_1, which is taken into account by firm 2, when it chooses price p_2. The price of the final good is $P = p_1 + p_2$.



$$\boldsymbol{A} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \end{array}$$

Figure 5: Example: two sequential monopolists

The standard method of finding the equilibrium in this game is backward induction. It starts by finding the best-response function of firm 2, by solving $\max_{p_2} p_2 D(p_1+p_2)$. The optimality condition is

$$\frac{d\pi_2}{dp_2} = D(p_1+p_2) + p_2 D'(p_1+p_2) = 0 \quad\Longleftrightarrow\quad e^{p_2}(1-p_2) = e^{-p_1}. \tag{3}$$

Solving this, gives best-response function $p_2^*(p_1) = 1 + W(e^{-(p_1+1)})$, where $W(\cdot)$ is the Lambert W function[16]. Substituting $p_2^*(p_1)$ to the optimization problem of firm 1 gives a maximization problem $\max_{p_1} p_1 D(p_1 + p_2^*(p_1))$. From this, we get the optimality condition

$$1 + e^{-(p_1+1)-W(e^{-(p_1+1)})} = p_1\left(1 + \frac{-e^{-(p_1+1)}W\big(e^{-(p_1+1)}\big)}{e^{-(p_1+1)} + W\big(e^{-(p_1+1)}\big)}\right). \tag{4}$$

Solving the equation numerically gives $p_1^* \approx 1.2088$, therefore $p_2^* \approx 1.0994$ and $P^* \approx 2.3082$. However, there is no analytic solution to optimality condition (4). This implies that the standard approach is intractable when the network is more complex than the

[15] Of course, not all demand functions satisfy assumption 3. In appendix B I discuss some such examples. t's worth noting that assumption 3 can have distinct implications: it might influence both the existence of an equilibrium and the characteristics of that equilibrium.

[16] Function $W(x)$ is defined as a solution to $x = W(x)e^{W(x)}$.

one studied here. The backward induction cannot be used, since computing best-response functions and substituting them to the maximization problems of other firms is not feasible. The issue is tractability—since the optimality conditions are non-linear, solving them leads to complex expressions. Replacing best-responses sequentially amplifies these complexities.

The solution to this problem comes from Hinnosaar (2023), which proposes characterizing the behavior of the following players by inverted best-response functions. The key observation is that although equation (3) is a highly non-linear function of p_1 and p_2 separately, fixing $P = p_1 + p_2$ leads to a linear equation for p_2 or equivalently $p_1 = P - p_2$. Therefore, for a fixed price of the final good, P, it is straightforward to find the price of firm 2 that is consistent with the final good price P. I denote this by $f_2(P)$, i.e.

$$f_2(P) = p_2 = -\frac{D(P)}{D'(P)} = g(P)$$

where $g(P) = 1 + e^{-P}$. Firm 1 knows that if it sets its price to p_1, the price of the final good will satisfy $P = p_1 + f_2(P)$. Therefore we can think of firm 1's problem as choosing P to solve $\max_P [P - f_2(P)]D(P)$. Taking the first-order condition gives us

$$f_1(P) = p_1 = P - f_2(P) = g(P)[1 - f_2'(P)].$$

Therefore if the final good's price in equilibrium is P^*, then optimal behavior of both players requires that $P^* = f_1(P^*) + f_2(P^*) = 2g(P^*) - g'(P^*)g(P^*)$. This equation is straightforward to solve, and the same argument can be easily extended to more players choosing sequentially.

There is one more pattern in these expressions that the characterization will exploit. Namely, the condition for equilibrium is

$$P^* = 2g(P^*) - g'(P^*)g(P^*) = 2g_1(P^*) + g_2(P^*),$$

where $g_1(P^*) = g(P^*)$ and $g_2(P^*) = -g_1'(P^*)g(P^*)$. The expression on the right-hand side consists of two elements. The first, $2g_1(P^*)$ captures the fact that there are two players who each individually maximize their profits. The second $g_2(P^*)$ captures the fact that player 1 influences player 2. It is straightforward to verify that for example if we would remove this influence, i.e., with two monopolists choosing their prices simultaneously, the equilibrium condition would become $P^* = 2g_1(P^*)$.

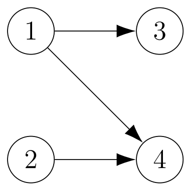
The main advantage of this approach is tractability. Instead of solving non-linear equations in each step and inserting the resulting expressions into the next maximization problems, which results in more complex non-linear expressions, this approach allows combining all necessary conditions of optimality into one necessary condition. Under assumptions 2 and 3, the resulting expression has a unique solution, which gives us a unique candidate for an interior equilibrium. Under the same assumptions, the sufficient conditions for optimality are also satisfied and therefore it determines unique equilibrium.

4.2 Example: Interconnected Decisions

The second example illustrates a new issue that arises in the case of networks—the decisions are interconnected. For example, on the network depicted by figure 4, firms L and

D make independent decisions, but due to their positions, they have different views on what happens before and after them. Firm D influences only F, but L influences T and C as well. Similarly, D takes p_R as given, whereas L does not observe p_R and therefore has to have an equilibrium conjecture about the optimal behavior of R. Therefore solving the game sequentially is not possible anymore.

To illustrate this issue, consider the simple network shown in figure 6. Let us assume that the demand is linear $D(P) = 1 - P$, and there are no costs and no price-takers.



$$\boldsymbol{A} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Figure 6: Example: network with interconnected decisions

The strategies of the firms are respectively p_1^*, p_2^*, $p_3^*(p_1)$, and $p_4^*(p_1, p_2)$. Let us first consider the problem of player 4, who observes p_1 and p_2 and expects equilibrium behavior from player 3. Therefore player 4 solves

$$\max_{p_4 \geq 0} p_4 \left[1 - p_1 - p_2 - p_3^*(p_1) - p_4\right],$$

which gives us a condition

$$p_4^*(p_1, p_2) = \frac{1}{2}\left[1 - p_1 - p_2 - p_3^*(p_1)\right].$$

While this condition provides a condition for the best-response function $p_4^*(p_1, p_2)$, we have not yet characterized it, as it would require knowing $p_3^*(p_1)$. Player 3 solves a similar problem, but does not observe p_2 and expects p_4 to be $p_4^*(p_1, p_2^*)$, that is

$$\max_{p_3 \geq 0} p_3 \left[1 - p_1 - p_2^* - p_3 - p_4^*(p_1, p_2^*)\right]$$

with the optimality condition

$$p_3^*(p_1) = \frac{1}{2}\left[1 - p_1 - p_2^* - p_4^*(p_1, p_2^*(p_1))\right].$$

Again, computing this best-response function explicitly, requires knowing $p_4^*(p_1, p_2)$, but also the equilibrium price of player 2, i.e., p_2^*. To compute the best-response functions explicitly (i.e., independently of each other), we first need to solve the equation system that we get by inserting p_2^* to the optimality condition of player 4. This gives us

$$p_3^*(p_1) = p_4^*(p_1, p_2^*) = \frac{1}{3}\left[1 - p_1 - p_2^*\right] \;\; \Rightarrow \;\; p_4^*(p_1, p_2) = \frac{1}{3}\left[1 - p_1\right] + \frac{1}{6}p_2^* - \frac{1}{2}p_2.$$

Note the prices p_3 and p_4 we have now characterized are still not the true best-response functions, since they depend on the equilibrium price p_2^*, which is yet to be determined.

For this we need to solve the problem of player 2, who expects player 1 to choose equilibrium price p_1^*

$$\max_{p_2 \geq 0} p_2 \left[1 - p_1^* - p_2 - p_3^*(p_1^*) - p_4^*(p_1^*, p_2)\right].$$

Taking the first-order condition and evaluating it at $p_2 = p_2^*$ gives a condition

$$\frac{1}{6}\left[2 - 2p_1^* - 5p_2^*\right] = 0. \tag{5}$$

Finally, player 1 solves a similar problem, taking p_2^* as fixed, i.e.

$$\max_{p_1 \geq 0} p_1 \left[1 - p_1 - p_2^* - p_3^*(p_1) - p_4^*(p_1, p_2^*)\right].$$

Again, taking the first-order condition and evaluating it at $p_1 = p_1^*$ gives

$$\frac{1}{3}\left[1 - 2p_1^* - p_2^*\right] = 0. \tag{6}$$

Solving the equation system equations (5) and (6) gives us $p_1^* = \frac{3}{8}$, $p_2^* = \frac{1}{4}$. Inserting these values to the functions derived above gives us the best-response functions $p_3^*(p_1) = \frac{1}{4} - \frac{1}{3}p_1$ and $p_4^*(p_1, p_2) = \frac{3}{8} - \frac{1}{3}p_1 - \frac{1}{2}p_2$. We can also compute the equilibrium prices $p_3^*(p_1^*) = p_4^*(p_1^*, p_2^*) = \frac{1}{8}$. Therefore equilibrium price of the final good is $P^* = \frac{7}{8}$.

As the example illustrates, finding the equilibrium strategies requires solving a combination of equation systems in parallel with finding the best-response functions. Each additional edge in the network can create a new layer of complexity.

The inverted best-response approach solves this issue as follows. Consider the optimization problem of firm 4. For given (p_1, p_2), it chooses optimal p_4. We can rethink its optimization problem as choosing the final good price $P = p_1 + p_2 + p_3^*(p_1) + p_4$ that it wants to induce. We can rewrite its maximization problem as

$$\max_{P} [P - p_1 - p_2 - p_3^*(p_1)]D(P).$$

We get the optimality condition

$$D(P) + [P - p_1 - p_2 - p_3^*(p_1)]D'(P) = 0 \iff P - p_1 - p_2 - p_3^*(p_1) = g(P) = 1 - P.$$

This is a necessary condition for optimality, but since the problem is quadratic, it is easy to see that it is also sufficient. This expression gives implicitly the best-response function $p_4^*(p_1, p_2)$. But more directly, the expression on the left-hand side is the optimal p_4 that is consistent with the final good price P and the optimal behavior of firm 4. Let us denote it by $f_4(P) = 1 - P$. The problem for the firm 3 is analogous and gives $f_3(P) = 1 - P$.

Now, consider firm 2. Instead of choosing p_2 it can again consider the choice of the final good price P. Since only firm 4 observes its choice (and thus chooses $p_4 = f_4(P)$ as a response to desired P), the firm 2's problem can be written as

$$\max_{P} [P - p_1^* - p_3^*(p_1^*) - f_4(P)]D(P),$$

which gives us the optimality condition

$$[1 - f_4'(P)]D(P) + [P - p_1^* - p_3^*(p_1^*) - f_4(P)]D'(P) = 0,$$

or equivalently, $f_2(P) = P - p_1^* - p_3^*(p_1^*) - f_4(P) = 2(1 - P)$. Analogous calculation for firm 1 gives $f_1(P) = 3(1 - P)$. Now, the equilibrium price P^* of the final good must be consistent with individual choices. Therefore we get a condition

$$P^* = \sum_{i=1}^{4} f_i(P^*) = 7(1 - P^*).$$

Solving this equation gives us $P^* = \frac{7}{8}$ and individual prices $p_1^* = 3(1 - P^*) = \frac{3}{8}$, $p_2^* = \frac{2}{8}$, and $p_3^* = p_4^* = \frac{1}{8}$.

Notice that the same calculations could be applied easily for non-linear demand functions, with some $g(P) = -\frac{D(P)}{D(P)}$. This would give us an equilibrium condition

$$P^* = \sum_{i=1}^{4} f_i(P^*) = 4g_1(P^*) + 3g_2(P^*),$$

where $g_1(P) = g(P)$ and $g_2(P) = -g_1'(P)g(P)$. This is again the same pattern that we saw in the previous example, since the number of players is 4 and the number of edges is 3. In the case of linear demand, $g_1(P) = g(P) = 1 - P$ and therefore $g_2(P) = 1 - P$.

This example illustrates the advantage of the inverted best-response approach. As the approach combines all necessary conditions into one, the issues of interconnected decisions are automatically mitigated.

4.3 Characterization

As illustrated by the examples above, it is useful to define functions $g_1, \dots, g_n$, which capture relevant properties of the demand function. They are defined recursively as

$$g_1(P) = g(P) = -\frac{D(P)}{D'(P)} \text{ and } g_{k+1}(P) = -g_k'(P)g(P). \tag{7}$$

As the discussion about monopoly profit maximization and the examples illustrated, $g_1(P)$ captures the standard concavity of the profit function, whereas $g_2(P)$ captures the direct discouragement effect when a firm observes the price of another firm. Functions $g_3, \dots, g_n$ play a similar role in describing higher-order discouragement effects.

Note also that the adjacency matrix $\boldsymbol{A}$ provides a convenient way to keep track of the number of direct and indirect influences. Multiplying the adjacency matrix with a column vector of ones, $\boldsymbol{A1}$, gives a vector with the number of edges going out from each player (i.e., the sum over columns). Similarly, $\boldsymbol{1}'\boldsymbol{A1}$ is the total number of edges on the network, i.e., the total number of direct influences. Multiplying the adjacency matrix by itself, i.e., $\boldsymbol{A}^2 = \boldsymbol{AA}$ gives a matrix that describes two-edge paths, i.e., element $a_{i,j}^2$ is the number of paths from i to j with one intermediate step. Similarly, $\boldsymbol{A}^k$ is the matrix that describes the number of all k-step paths from each i to each j. When we take $k = 0$, then $\boldsymbol{A}^0$ is an identity matrix, which can be interpreted as 0-step paths (clearly, the only player that can be reached from player i by following 0 edges is player i himself). To simplify the notation, I assume that $\boldsymbol{A}^0$ is the identity matrix even when $\boldsymbol{A} = \boldsymbol{0}$.

Therefore $\boldsymbol{A}^k\boldsymbol{1}$ is a vector whose elements are the numbers of k-step paths from player i, which can be directly computed as $\boldsymbol{e}_i'\boldsymbol{A}^k\boldsymbol{1}$, where $\boldsymbol{e}_i$ is a column vector, where ith element is 1 and other elements are zeros. Similarly, $\mathbf{1}'\boldsymbol{A}^k\mathbf{1}$ is the number of all k-step paths in the network. The following expression makes these calculations for the network described by figure 4, which has six players, six edges, and one two-edge path ($R \to D \to F$).

$$
\begin{array}{c}
\\ L \\ T \\ F \\ C \\ D \\ R \\ \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}
\end{array}
\begin{array}{c}
\begin{array}{cccc} \boldsymbol{A}^0\mathbf{1} & \boldsymbol{A}^1\mathbf{1} & \boldsymbol{A}^2\mathbf{1} & \boldsymbol{A}^3\mathbf{1} \end{array} \\
\left[\begin{array}{cccc}
1 & 3 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 2 & 1 & 0 \\
6 & 6 & 1 & 0
\end{array}\right]
\end{array}. \tag{8}
$$

With this notation, I can now state the main result of this paper, the characterization theorem that states that there exists a unique equilibrium and shows how it is characterized using the components we have discussed.

Theorem 1. *There is a unique equilibrium, the final good price P^* is the solution to*

$$P^* - C = \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1} g_k(P^*), \tag{9}$$

and the individual prices are $p_i^* = c_i + \sum_{k=1}^{m} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1} g_k(P^*)$ for all i.

The proof in appendix A builds on the ideas discussed above. A few remarks are in order. The uniqueness is straightforward to establish. Assumption 3 implies that each $g_k(P)$ is non-negative and weakly decreasing (this is formally shown in lemma 2 in appendix A). The right-hand side of equation (9) is therefore decreasing, whereas the left-hand side is strictly increasing.[17] Connection to inverted best-response functions is also clear, as the individual prices are determined by $p_i^* = c_i + f_i(P^*)$.

5 Multiple-marginalization Problem

Let me first interpret the equilibrium condition equation (9) by comparing it with the known benchmark cases. First, when all firms are price-takers ($m = 0$), then the network is empty and therefore the right-hand side of (9) is zero. As expected, the equilibrium condition is, therefore, $P^* = C$, i.e., the price of the final good equals the marginal cost of the final good. Standard arguments imply that this is also the welfare-maximizing solution.

Second, suppose that there is a single monopolist, i.e., $m = 1$ and $\boldsymbol{A} = [0]$. Therefore there is a single element on the right-hand side of (9) with value $g_1(P^*)$. We can rewrite the condition as

$$\frac{P^* - C}{P^*} = \frac{g_1(P^*)}{P^*} = \frac{1}{\varepsilon(P^*)}, \tag{10}$$

[17] If assumption 3 fails, equation (9) may not have a solution or may have multiple solutions. Moreover, without assumption 3, equation (9) is only necessary but not sufficient for equilibrium. In appendix B, I discuss some such examples.

which is the standard inverse-elasticity rule: mark-up (Lerner index) equals the inverse elasticity. There is a usual monopoly distortion—as the monopolist does not internalize the impact on the consumer surplus, the price equilibrium price of the final good is higher and the equilibrium quantity lower than the social optimum. This is also the joint profit-maximization outcome.

Third, consider $m > 1$ monopolists who are making their decisions simultaneously. That is, the network has m nodes but no edges. Analogously with a single monopolist, we can then rewrite the equilibrium condition as

$$\frac{P^* - C}{P^*} = \mathbf{1}'\boldsymbol{A}^0\mathbf{1}\frac{g_1(P^*)}{P^*} = \frac{m}{\varepsilon(P^*)} > \frac{1}{\varepsilon(P^*)}. \tag{11}$$

The total markup is now strictly higher than in the case of a single monopolist. This is the *multiple-marginalization problem*—firms do not internalize not only the impact on consumer surplus but also the impact on the other firms. Therefore the distortion is even larger than in the case of a single monopolist, which means that both the total profits and the social welfare are reduced compared to a single monopolist.[18]

The novel case studied in this paper is with multiple monopolists and some influences. That is $m > 0$ and $\boldsymbol{A} \neq \mathbf{0}$. In this case, the condition can be written as

$$\frac{P^* - C}{P^*} = \frac{m}{\varepsilon(P^*)} + \sum_{k=2}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}\frac{g_k(P^*)}{P^*}. \tag{12}$$

The total markup and therefore the distortion is even higher than with m independent monopolists. The intuition for this is simple: suppose that there is a single edge, so that firm i influences firm j. Then, in addition to the trade-offs firm i had before, raising the price now will reduce the profitability of firm j, who will respond by reducing its price. Therefore p_i will be higher and p_j lower than with simultaneous decisions. How about the price of the final good, which depends on the sum of p_i and p_j? If the reduction in p_j would be so large that the total price does not increase, then p_i would not be optimal, as the profit of firm i is $(p_i - c_i)D(P)$, i.e., increasing in p_i and decreasing P, so i would want to raise the price even further. Thus in equilibrium, it should be that the price of the final good is increased. I formalize and generalize this observation as corollary 1. The corollary follows from equation (9) and non-negativity of g_k functions.

Corollary 1 (Magnified Multiple-marginalization Problem)**.** *Suppose that there are two networks $\boldsymbol{A}$ and $\boldsymbol{B}$ such that*

1. $\mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1} \geq \mathbf{1}'\boldsymbol{B}^{k-1}\mathbf{1}$ *for all $k \in \{1, \dots, m\}$ and*
2. $\mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1} > \mathbf{1}'\boldsymbol{B}^{k-1}\mathbf{1}$ *for at least one k,*

then both social welfare and total profit in the case of $\boldsymbol{A}$ is lower than with $\boldsymbol{B}$.

[18] There are two ways to think about multiple-marginalization. Since the seminal work by Spengler (1950), it has been mostly presented as a problem of sequential pricing. However, Cournot already observed almost 200 years earlier that two monopolists pricing perfect complements independently would distort the allocation more than a single monopolist pricing both (Sonnenschein, 1968). In this paper, I analyze both. I refer to the Cournot interpretation as multiple-marginalization and the Spengler version as magnified multiple-marginalization.

The result shows that the multiple-marginalization problem is increased with strategic influences but does not give a magnitude for it. To illustrate that the impact may be severe, let me give some numeric examples. First, suppose that demand is linear, $D(P) = 1 - P$, there are no costs, and there are no price-takers. Standard calculations imply that the maximized total welfare would be $\frac{1}{2}$ and a single monopolist would choose price $\frac{1}{2}$, which would lead to a dead-weight loss of $\frac{1}{8}$. Therefore with any network, the dead-weight loss is at least $\frac{1}{8}$ and at most $\frac{1}{2}$. Figure 7 illustrates the difference between the dead-weight loss in the best case (simultaneous decisions) and the worst case (sequential decisions). Even in the best case (blue line with triangles), the multiple-marginalization problem can be severe and is increasing in m. However, the distortions with strategic interactions (red line with circles) are much higher for any m, and the dead-weight loss approaches to full destruction of the social welfare quickly. This comparison shows that strategic influences magnify the multiple-marginalization problem with any m.

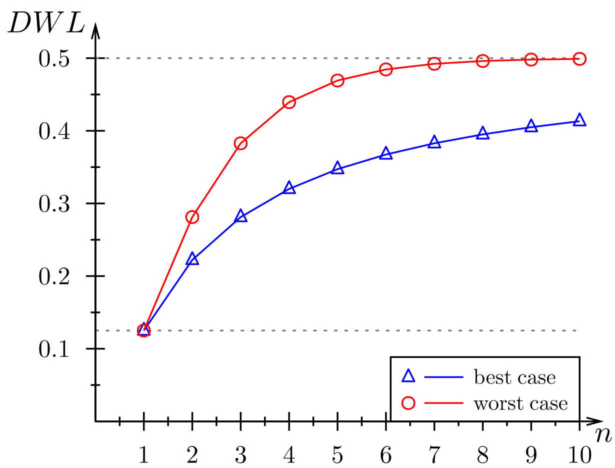
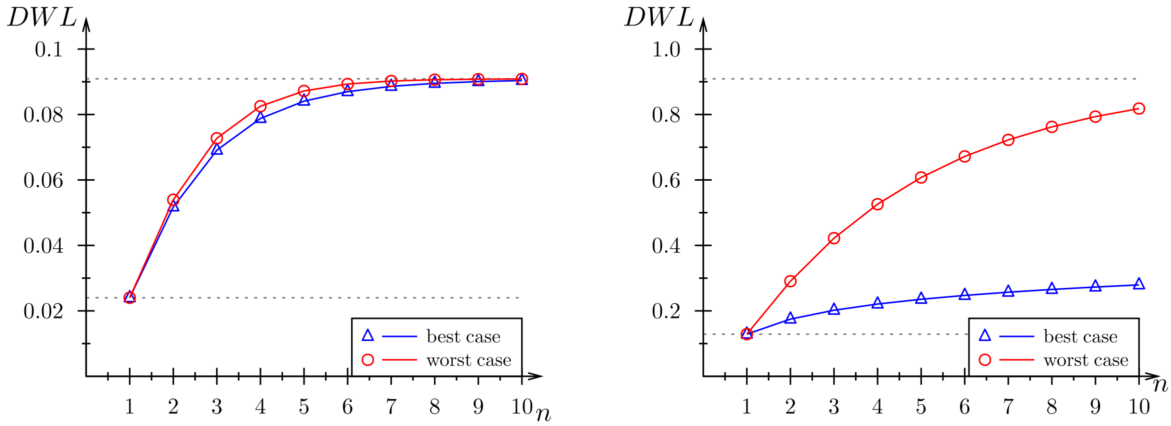


Figure 7: Example: comparison of dead-weight loss in the model with linear demand between the best case (simultaneous decisions) and the worst case (sequential decisions)

How much the number of firms matters compared to strategic influences depends on the shape of the demand function. This is illustrated by figure 8, which provides the same comparison with a more general power demand function $D(P) = \sqrt[\beta]{1-P}$.[19] When $\beta < 1$, then relatively high weight is given to more direct influences. Indeed, figure 8a shows that when $\beta = \frac{1}{10}$ then the best-case and the worst-case dead-weight loss do not differ much. On the other hand, when $\beta > 1$ the weight is larger on more indirect influences. This is illustrated by figure 8b, where the difference between the two cases is large.[20]

[19] Generally $DWL(P^*) = \int_0^{P^*} [D(P) - D(P^*)]dP$. The calculations and the impact of β in the case of power demand function are discussed in more detail in the next section.

[20] Even more generally, the relative sizes of $g_1(P^*), \dots, g_k(P^*)$ capture the relative weights on direct and indirect influences. If $g_k(P^*) \ll g_1(P^*)$ for all k, then the network structure does not play a big role. Whereas, in the other extreme with $g_k(P^*) \gg g_1(P^*)$, the magnification effect is large.



(a) $\beta = \frac{1}{10}$ (b) $\beta = 10$

Figure 8: Examples: comparison of dead-weight loss with power demand $D(P) = \sqrt[\beta]{1-P}$ between the best case (simultaneous decisions) and the worst case (sequential decisions)

6 Influentiality

6.1 A Measure of Influentiality

All monopolists on the network have some market power and therefore earn strictly positive profits. But some firms are more influential than others. Which ones and how does this depend on the network? The answer comes directly from the characterization in theorem 1. For brevity, let me denote

$$I_i(\boldsymbol{A}) = \sum_{k=1}^{m} \boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1} g_k(P^*), \tag{13}$$

which is a sum of scalars $\boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1}$ weighted by $g_k(P^*)$. Note that $\boldsymbol{e}_i' \boldsymbol{A}^0 \mathbf{1} = 1$, $\boldsymbol{e}_i' \boldsymbol{A}^1 \mathbf{1}$ is the number of players i influences, $\boldsymbol{e}_i' \boldsymbol{A}^2 \mathbf{1}$ is the number of two-edge paths starting from i, and so on. Therefore $I_i(\boldsymbol{A})$ can be interpreted as a measure of influentiality of player i.

Fixing the equilibrium price of the final good P^*, the individual markups are $p_i^* - c_i = I_i(\boldsymbol{A})$ and therefore profits $\pi_i(\boldsymbol{p}^*) = (p_i^* - c_i) D(P^*) = I_i(\boldsymbol{A}) D(P^*)$. Therefore $I_i(\boldsymbol{A})$ fully captures the details of the network that affect firm i's action and payoff. Corollary 2 provides a formal statement.

Corollary 2 ($I_i(\boldsymbol{A})$ Summarizes Influences)**.** *$I_i(P^*) > I_j(P^*)$ if and only if $\pi_i(\boldsymbol{p}^*) > \pi_j(\boldsymbol{p}^*)$ and $p_i - c_i > p_j^* - c_j$.*

This measure of influentiality $I_i(\boldsymbol{A})$ depends both on the network structure and the demand function. There are some cases when we can say more. In particular, if firm i has more influences in all levels than firm j, i.e. $\boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1} \geq \boldsymbol{e}_j' \boldsymbol{A}^{k-1} \mathbf{1}$ for all k and the inequality is strict for at least one k, then $I_i(\boldsymbol{A}) \geq I_j(\boldsymbol{A})$ regardless of the weights $g_k(P^*)$. The inequality is strict whenever $g_k(P^*) > 0$ for k such that $\boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1} > \boldsymbol{e}_j' \boldsymbol{A}^{k-1} \mathbf{1}$. For example, when firm i influences firm j, then $\boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1} \geq \boldsymbol{e}_j' \boldsymbol{A}^{k-1} \mathbf{1}$ and the inequality is strict for at least $k = 1$, so $I_i(\boldsymbol{A}) > I_j(\boldsymbol{A})$ with any demand function.

6.2 Connections with Network Centrality Measures

The measure of influentiality defined above is reminiscent of the classic measures of centrality as they capture the same effects: a player is more influential if it influences either more players or more influential players. The difference is that while the classic centrality measures are defined purely using network characteristics, the influentiality measure defined here has endogenous weights that are determined by the model parameters such as the demand function, costs, and also by the price of the final good.

In some special cases, the connection is even closer. Consider the case of power demand $D(P) = d\sqrt[\beta]{a - bP}$. As discussed in section 7.2, it implies linear $g_k(P) = \beta^k(\overline{P} - P)$. Therefore $I_i(\boldsymbol{A}) = (\overline{P} - P^*)B_i^\beta(\boldsymbol{A})$, where $B_i(\boldsymbol{A})^\beta = \sum_{k=1}^m \beta^k \boldsymbol{e}_i' \boldsymbol{A}^{k-1}\mathbf{1}$ is the Bonacich centrality measure of player i. The general measure $I_i(\boldsymbol{A})$ can be thought of as a generalization of Bonacich centrality where the weights are endogenously determined by the demand function and the equilibrium, rather than having exponential decay β^k.[21] Linear demand $D(P) = a - bP$ is a special case of power demand with $\beta = 1$. Therefore the influentiality measure $I_i(\boldsymbol{A})$ simplifies to the Bonacich centrality measure with $\beta = 1$, i.e., equal weight for each level of influences.

However, the influence measure does not always have to have a flavor of Bonacich centrality. Let me provide two more examples to show this. First, suppose $D(P) = de^{\sqrt{2(a-bP)/b}}$. This a specifically constructed demand function, which implies $g(P) = g_1(P) = \sqrt{2(a - bP)}$ and therefore $g_2(P) = b$, which means that $g_k(P) = 0$ for all $k > 2$. With these weights the influentiality measure simplifies to $I_i(\boldsymbol{A}) = \sqrt{2(a - bP^*)} + b\boldsymbol{e}_i'\boldsymbol{A}\mathbf{1}$, i.e., depends only on the number of players directly influenced by player i. That is, the influentiality measure is a linear function of the degree centrality in this case.

For another example, consider logit demand $D(P) = d\frac{e^{-\alpha P}}{1+e^{-\alpha P}}$. As I will show in section 7.3, it may lead to complex expressions, but when m is large enough, then $g_1(P^*) \approx \frac{1}{\alpha}$ and $g_k(P^*) \approx 0$ for $k > 1$. Therefore $I_i(\boldsymbol{A}) \approx \frac{1}{\alpha}$. This means that in the case of logit demand with sufficiently many players, the network structure does not affect the pricing of the individual firms. The relevant centrality measure is approximately a constant. These observations are summarized by table 1.

Demand $D(P)$	Influentiality $I_i(\boldsymbol{A})$	Equivalent Network Centrality Measure
Power $d\sqrt[\beta]{a - bP}$	$\left(\frac{a}{b} - P^*\right) B_i^\beta(\boldsymbol{A})$	$B_i^\beta(\boldsymbol{A})$ = Bonacich centrality (with β)
Linear $a - bP$	$\left(\frac{a}{b} - P^*\right) B_i(\boldsymbol{A})$	$B_i(\boldsymbol{A})$ = Bonacich centrality with $\beta = 1$
$de^{\sqrt{2(a-bP)/b}}$	$\sqrt{2(a - bP^*)} + bD_i(\boldsymbol{A})$	$D_i(\boldsymbol{A}) = \boldsymbol{e}_i'\boldsymbol{A}\mathbf{1}$ = Degree centrality
Logit $d\frac{e^{-\alpha P}}{1+e^{-\alpha P}}$	$\to \frac{1}{\alpha}$	Approximately a constant

Table 1: Examples of demand functions with which the measure of influentiality $I_i(\boldsymbol{A})$ simplifies to one of the standard network centrality measures

[21]The standard definition of Bonacich centrality requires $\beta < 1$, otherwise the sum may not converge and the measure would not be well-defined. As the influence networks here are acyclic, any $\beta > 0$ is allowed. Values $\beta \geq 1$ arise whenever $D(P) = d\sqrt[\beta]{a - bP}$, which is a quite natural assumption for a demand function.

7 Computing the Equilibrium

In this subsection, I show how the equilibrium characterization could be used to compute the equilibrium and study some of the most common demand functions where the characterization is even simpler.

7.1 Linear Demand

Suppose that the demand function is linear $D(P) = a - bP$. Then $g(P) = -\frac{D(P)}{D'(P)} = \overline{P} - P = g_1(P)$ with $\overline{P} = \frac{a}{b}$ and therefore for all $k > 1$, $g_{k+1}(P) = -g_k'(P)g(P) = \overline{P} - P$. Equation (9) simplifies to

$$P^* - C = \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) = (\overline{P} - P^*)B(\boldsymbol{A}), \tag{14}$$

where $B(\boldsymbol{A}) = \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}$ is the sum of the number of influences of all levels, i.e., the number of players ($\mathbf{1}'\boldsymbol{A}^0\mathbf{1} = m$) plus the number of edges plus the number of two-edge paths, and so on. Equation (14) is a linear equation and its solution is the equilibrium price

$$P^* = \frac{C + \overline{P}B(\boldsymbol{A})}{1 + B(\boldsymbol{A})}. \tag{15}$$

As we would expect, increasing costs and increasing demand ($\overline{P} = \frac{a}{b}$ in particular) will raise the equilibrium price, but the pass-through is imperfect. Increasing the number of firms or the number of connections between firms increases the equilibrium price through the marginalization effects discussed above. Similarly, we can compute the markups for individual firms,

$$p_i^* = c_i + \sum_{k=1}^{m} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) = c_i + \frac{B_i(\boldsymbol{A})}{1 + B(\boldsymbol{A})}(\overline{P} - C), \tag{16}$$

where $B_i(\boldsymbol{A}) = \sum_{k=1}^{m} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}$ is the sum of influences of firm i, i.e., $\boldsymbol{e}_i'\boldsymbol{A}^0\mathbf{1} = 1$ ("influencing" oneself) plus $\boldsymbol{e}_i'\boldsymbol{A}^1\mathbf{1}$ = number of players i influences plus the number of paths starting from i. By construction $B(\boldsymbol{A}) = \sum_{i=1}^{m} B_i(\boldsymbol{A})$.

Consider the example of network described by figure 4, for which the corresponding $\boldsymbol{A}^{k-1}\mathbf{1}$ terms are computed in equation (8). Suppose that $D(P) = 1 - P$, and there are no costs and no price-takers ($C = 0$). Then $B(\boldsymbol{A}) = 6 + 6 + 1 = 13$ and therefore $P^* = \frac{B(\boldsymbol{A})}{1+B(\boldsymbol{A})} = \frac{13}{14}$. Similarly, individual prices $p_i^* = \frac{B_i(\boldsymbol{A})}{1+B(\boldsymbol{A})}$. For example, $p_L^* = \frac{4}{14}, p_T^* = p_F^* = p_C^* = \frac{1}{14}, p_D^* = \frac{2}{14}$, and $p_R^* = \frac{4}{14}$. In particular, observe that $p_L^* = p_R^*$, but for different reasons—firm L influences three firms directly, whereas R influences two firms directly and one indirectly. In the case of linear demand, these two types of influences are weighted equally.

7.2 Power Demand

The calculations are similar for more general power demand $D(P) = d\sqrt[\beta]{a - bP}$. Then $g(P) = \beta(\overline{P} - P)$ with $\overline{P} = \frac{a}{b}$ and therefore $g_k(P) = \beta^k(\overline{P} - P)$, so that equation (9) gives the same expression for the equilibrium price of the final good

$$P^* = \frac{C + \overline{P} B^\beta(\boldsymbol{A})}{1 + B^\beta(\boldsymbol{A})} \tag{17}$$

but now $B^\beta(\boldsymbol{A}) = \sum_{k=1}^m \beta^k \mathbf{1}' \boldsymbol{A}^{k-1} \mathbf{1}$, i.e., the influences in various levels are weighted by $1, \beta, \beta^2, \ldots$. Then β can be interpreted as decay or discount factor for more indirect influences.[22] Similarly, for individual firms,

$$p_i^* = c_i + \frac{B_i^\beta(\boldsymbol{A})}{1 + B^\beta(\boldsymbol{A})} (\overline{P} - C), \tag{18}$$

where $B_i^\beta(\boldsymbol{A}) = \sum_{k=1}^m \beta^k \boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1}$, i.e., influences are again weighted by factor β^k.

Consider the example from figure 4 again, with $C = 0$, and demand function $D(P) = \sqrt[\beta]{1 - P}$. In particular, if $\beta = \frac{1}{2}$, then $B^{\frac{1}{2}}(\boldsymbol{A}) = 6 + \frac{1}{2} 6 + \frac{1}{4} 1 = \frac{37}{4}$, so $P^* = \frac{37}{41}$. As anticipated above, since higher weight is on direct influences than indirect influences, firm L sets a higher price (and earns higher profit) than firm R, $p_L^* = \frac{10}{41} > p_R^* = \frac{9}{41}$. This is also the reason why the difference between the worst-case and the best-case on figure 8a was relatively small. On the other hand, $\beta = 2$ implies $P^* = \frac{22}{23}$ and $p_L^* = \frac{7}{23} < p_D^* = \frac{9}{23}$, since now the weight is higher on indirect influences. This explains the larger difference in figure 8b.

7.3 Logit Demand

Take logit demand $D(P) = d \frac{e^{-\alpha P}}{1 + e^{-\alpha P}}$ with $\alpha > 0$. Then $g(P) = \frac{1}{\alpha} \left[1 + e^{-\alpha P}\right]$.

Let us first consider the example discussed in previous subsections to illustrate how the characterization result could be used for more complicated demand functions. Suppose again that $C = 0$ and the network is the one described by figure 4. Since the depth of the network is $d(\boldsymbol{A}) = 3$, we need to compute functions

$$\begin{aligned}
g_1(P) &= g(P) = \frac{1}{\alpha} \left[1 + e^{-\alpha P}\right], \\
g_2(P) &= -g_1'(P) g(P) = \frac{1}{\alpha} \left[1 + e^{-\alpha P}\right] e^{-\alpha P}, \\
g_3(P) &= -g_2'(P) g(P) = \frac{1}{\alpha} \left[1 + e^{-\alpha P}\right] e^{-\alpha P} \left[1 + 2e^{-\alpha P}\right].
\end{aligned}$$

The equilibrium condition (9) takes the form $P^* = 6g_1(P^*) + 6g_2(P^*) + g_3(P^*)$, which is straightforward to solve numerically. For example, when $\alpha = 1$, we get

$$P^* = 6 + 13e^{-P^*} + 9e^{-2P^*} + 2e^{-3P^*},$$

which implies $P^* \approx 6.0313$ and individual prices $p_L^* \approx 1.0096, p_T^* = p_F^* = p_C^* \approx 1.0024, p_D^* \approx 1.0048$, and $p_R^* \approx 1.0096$.

The numeric results point to a more specific property of the equilibrium behavior in the case of logit demand. Namely, all prices are only slightly above 1. Inspecting

[22] Note that $\beta > 0$ (as otherwise demand would not be decreasing), but it can be bigger or smaller than 1. In fact, when $\beta = 1$ the demand function is linear, so that $B^\beta(\boldsymbol{A}) = B(\boldsymbol{A})$.

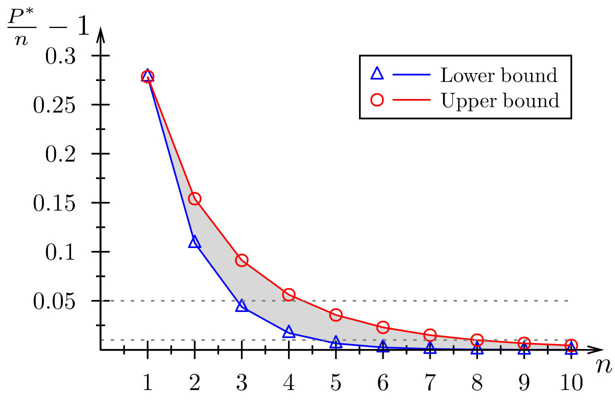


Figure 9: Bounds for equilibrium prices with logit demand $D(P) = \frac{e^{-P}}{1+e^{-P}}$ and $C = 0$ depending on the number of firms.

$g_k(P)$ functions above reveals the reason. Namely, the term $e^{-\alpha P^*}$ converges to zero as P^* increases. Therefore, for sufficiently large P^*, the weight $g_1(P^*)$ converges to a constant $\frac{1}{\alpha}$, whereas the weights $g_k(P^*)$ for $k > 1$ converge to zero. Therefore, if the equilibrium price P^* is large enough, it is almost fully driven by the number of players. This observation is formalized as the following lemma 1.

Lemma 1 (Approximate Equilibrium with Logit Demand). *With logit demand $D(P) = d\frac{e^{-\alpha}}{1+e^{-\alpha P}}$, the price of final good P^* and individual prices p_i^* satisfy the following conditions*

1. *$P^* > C + \frac{m}{\alpha}$ and $p_i^* > c_i + \frac{1}{\alpha}$ for all i,*
2. *$P^* = C + \frac{m}{\alpha} + O\left(\left[\frac{2}{e}\right]^m\right)$ and $p_i^* = c_i + \frac{1}{\alpha} + O\left(\left[\frac{2}{e}\right]^m\right)$ for all i.*[23]

Lemma 1 implies when m is large enough, then $P^* \approx C + \frac{m}{\alpha}$ and each $p_i^* \approx c_i + \frac{1}{\alpha}$. This is a limit result, but as we saw from the example above, the approximation with $m = 6$ seems already quite precise. Figure 9 illustrates that the convergence is indeed fast. It shows that while for small numbers of players, there is a difference between the lower bound (simultaneous decisions) and the upper bound (sequential decisions), the difference shrinks quickly and becomes negligible with 5–10 players. In particular, the figure illustrates that $\frac{1}{m}\left[P^* - C - \frac{m}{\alpha}\right] \approx 0$ for any network with about ten players or more.

8 Discussion

This paper characterizes the equilibrium behavior for a general class of price-setting games on a network. Under regularity assumptions, there is a unique equilibrium, which is straightforward to compute even with non-linear demands functions and complex networks. For the most common demand functions, such as linear, power, and logit demand, I provide even simpler characterization results.

[23]Where $f(m) = O(g(m))$ means that $\limsup_{m\to\infty} \left|\frac{f(x)}{g(x)}\right| < \infty$.

The key distortion is the multiple-marginalization, which leads to too high markups both for efficiency and joint profit maximization. The marginalization problem increases with the number of firms but is magnified by strategic interactions. Firms set too high markups, not only because they do not internalize the negative impact on consumer surplus and other firms' profits, but also because they benefit from discouraging the other firms from setting high markups.

The results define a natural measure of influentiality that ranks firms according to their markups and profits. Firms are more influential if they influence more firms or more influential firms. In some special cases, the influentiality measure simplifies to standard measures of centrality. I give examples where it takes the form of Bonacich centrality, degree centrality, or is independent of the network structure.

The results of this paper are quite general in terms of network structure and demand functions, but do I make several simplifying assumptions in other areas. For example, I assume constant marginal costs, which provides the model with tractability. Additionally, I model the competition in an extreme way, with firms either being monopolists or price-takers. This covers some intermediate cases, where firms may have market power in certain price ranges but become price-takers at higher prices. It would be interesting to study intermediate forms of imperfect competition. It would be natural to expect that a market power for an oligopolistic supplier of an input is between the two extremes.

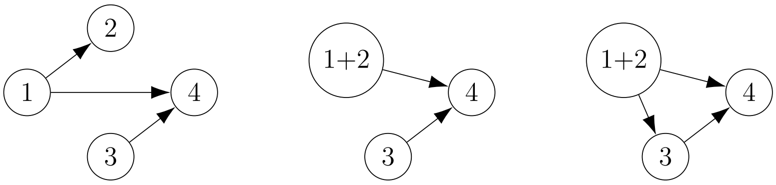
In this paper, the analysis is described in terms of price setting on a supply-chain network that supplies a single final product. There are other applications fitting the same mathematical model. An obvious example is multiple monopolists sell perfect complements. More generally, the model applies whenever multiple players choose actions, so that their payoffs depend linearly on their own actions, the marginal benefit is a decreasing function of the total action, and the actions are (higher-order) strategic substitutes. For example, the private provision of public goods and contests satisfy this general description.

Finally, the results have significant policy implications, which I have not discussed so far. In the following, I describe two simple examples to highlight two important policy implications. First, when analyzing mergers and acquisitions, it is crucial for the regulator to consider how the network of influences is affected. Second, in trade policy, small increases in any tariffs typically hurt all players, but more influential firms are harmed the most. Moreover, when considering non-marginal changes in trade policy, it is again crucial to consider the impact on the network as a whole.

8.1 Example: Mergers

To highlight how merger policy can be affected by changes in the network of influences, consider the following example with four monopolists. Firm 1 produces a raw material that is an input for two intermediate good producers, firms 2 and 3. The final good producer firm 4 uses inputs from firms 1, 2, and 3 to produce the final good and sell it to the final consumer directly. Let us assume that before the merger firms 2 and 4 are influenced by firm 1. Firm 3 makes a choice independently of firm 1 and influences firm 4. This is illustrated by figure 10a. To make the example more concrete, suppose that there are no costs and the demand function is $D(P) = \sqrt[\beta]{1-P}$ with $\beta > 0$, which allows us to use explicit formulas from section 7.2.

Suppose now that firms 1 and 2 would like to merge so that the new form (called



(a) Pre-merger (b) Scenario A (c) Scenario B

Figure 10: Examples: merger scenarios for firms 1 and 2.

$1 + 2$) would produce both the raw material and the first intermediate good. Should the competition authority approve the merger? There are a few important aspects that the policymaker may consider. First, how does it affect the competition? By assumptions, we are analyzing the production of a single product with fixed demand function and monopolistic input providers, so the competition remains unaffected. Second, does it lead to cost reductions or synergies? Again, we assume that there are no costs, so this remains unaffected. Third, there is one less firm, which reduces the marginalization problem. Combining these arguments, conventional wisdom suggests that the merger is socially desirable.

However, notice that the network of influences is necessarily changed as firm $1 + 2$ now makes a joint profit maximization decision. The key question is, what does the new network of influences look like? Let us consider two scenarios.

Scenario A: after the merger, the new firm $1 + 2$ simply internalizes all the influences within the former firms 1 and 2 and interactions with the remaining firms 3 and 4 are unchanged (figure 10b). It is easy to see that due to strict reduction of marginalization, the price of the final good goes down, which means that both the consumer surplus and the total profit of all firms strictly increase. But the profit of the firm $1+2$ is higher than the individual profits of firms 1 and 2 if and only if β is low enough ($\beta \leq \overline{\beta} \approx 0.4256$). The merger increases the total profit of all firms but reduces the influentiality of firms 1 and 2, who are then able to capture a smaller share of the profits. They are only willing to merge when the first effect dominates. We can conclude that it is a socially desirable merger but does not happen unless β is small enough.[24] Suppose a regulator considers a request for permission from firms 1 and 2 to merge and the estimated demand function has $\beta > \overline{\beta}$. In that case, this is might not the right scenario to consider.

Scenario B: Suppose alternatively that after the merger, the new firm $1 + 2$ becomes more influential, so that it can also influence the decision of firm 3, as illustrated by figure 10c. Now there are three direct influences as in the pre-merger scenario. Importantly, there is one less firm but one more indirect influence ($1 + 2 \to 3 \to 3$), which means that the price of the final good decreases if and only if $\beta \leq 1$. Otherwise, this new merger is not socially desirable. However, it is straightforward to verify that the joint profits of

[24]This scenario is inspired by Salant et al. (1983), who showed that in Cournot oligopoly, the total profit may increase if some firms merge, while the profits of the merging firms are decreasing. As they did not consider sequential choices, the indirect effects considered here were not applicable, while here they may change the conclusions.

firms 1 and 2 are lower than the profit of the new firm $1+2$ for all β. Therefore this is a merger that would always happen but is socially desirable only if β is not too large. For example, if the regulator has a good reason to believe that $\beta > 1$, such a request to merge should be denied, which contradicts the conventional wisdom discussed above.

8.2 Example: Trade

The previous discussion already illustrates the importance of taking the changes in the network of influences into account when considering policy decisions. The same message applies to trade policies as well. Changes in tariffs or quotas and any trade restrictions influence the supply chains and the interactions of firms within a supply chain. Therefore they naturally impact the network of influences. The results in this paper provide a tool to compare the outcomes under different scenarios.

In the simplest case, when changes in tariffs are marginal so that the network of influences is unchanged, theorem 1 provides specific predictions. In particular, let us assume that the marginal cost of each input is $c_i = \hat{c}_i + t_i$, where $\hat{c}_i$ is the physical cost and t_i is the tariff on good i. Then changes in tariffs can be thought of as changes in $\boldsymbol{c} = (c_0, c_1, \ldots, c_n)$.[25]

A surprising implication of theorem 1 is that when the changes are marginal, only the total marginal cost $C = \sum_{i=0}^{m} c_i$ affects the equilibrium price, profits, consumer surplus, and total welfare. Moreover, the equilibrium price is increasing, and all payoffs are decreasing with C. This is easy to see from equation (9). Differentiating the equation gives

$$P^* - \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) = C \quad \Rightarrow \quad \frac{dP^*}{dC} = \frac{1}{1 - \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k'(P^*)} > 0. \tag{19}$$

as each $g_k'(P^*) \leq 0$. Therefore only the aggregate changes in tariffs affect the equilibrium price of the final good. Clearly, consumer surplus depends only on the price of the final good. Although individual prices p_i^* are affected by individual tariffs, the individual equilibrium profits $\pi_i(\boldsymbol{p}^*) = I_i(\boldsymbol{A})D(P^*)$ are only affected by tariffs through their impact on the price of the final good. Any increase in the sum of tariffs leads to an increase in the price of the final good and therefore decrease in profits that are proportional to influentiality measure $I_i(\boldsymbol{A})$. So, the more influential firms are affected more by the tariffs, regardless of which individual tariffs or subsidies are imposed. Finally, defined as the sum of consumer surplus, all profits, and the tariff revenues are marginally affected as

$$TW = \underbrace{\int_{P^*}^{\infty} D(P)dP}_{=\text{Consumer Surplus}} + \underbrace{D(P^*)(P^* - \hat{C} - T)}_{=\sum_i \pi_i^*} + D(P^*)T. \tag{20}$$

The direct effects of tariffs cancel out as profits are reduced exactly by the tariff revenue. The only effect is through the change in the price of the final good, which is increasing

[25] In this simple example, I do not take into account many crucial elements of trade policies, such as the redistribution of profits between domestic and foreign entities or even different countries. It would be more accurate to refer to this exercise as an examination of changes in marginal taxes. However, for the sake of interpretive clarity, I will continue to refer to them as tariffs.

in tariffs. Differentiating total welfare with respect to the equilibrium price of the final good gives

$$\frac{dTW}{dP^*} = -D(P^*) + D'(P^*)(P^* - \hat{C}) + D(P^*) = D'(P^*)(P^* - \hat{C}) < 0 \iff P^* > \hat{C}. \quad (21)$$

This is a standard textbook finding implying that the socially optimal tax on a monopoly is, in fact, a subsidy that equalizes price with marginal cost.

Note that there are important aspects missing from this simple application of the model. The main goal of using tariffs and other trade policies is to affect trade flows. This changes the supply chain network and the network of influences. As highlighted in the discussion above, it may have a large impact both on the consumer surplus and the profits and should be therefore considered in any such policy evaluation.

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A Proofs

A.1 Proof of proposition 1

Proof. In each case, I directly verify the claim:

1. Linear demand is a special case of power demand with $d = \beta = 1$.

2. Power demand implies $g(P) = -\frac{d(a-bP)^{\frac{1}{\beta}}}{d\frac{1}{\beta}(a-bP)^{\frac{1}{\beta}-1}(-b)} = \beta(\overline{P} - P)$, where $\overline{P} = \frac{a}{b}$. Then $-g'(P) = \beta > 0$ and $(-1)^k \frac{d^k g(P)}{dP^k} = 0$, for all $k > 1$.

3. Logit demand implies $g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right]$. Then $(-1)^k \frac{d^k g(P)}{dP^k} = \alpha^{k-1}e^{-\alpha P} > 0$.

4. Exponential demand implies $g(P) = \frac{1}{\alpha}\left[\overline{P}e^{-\alpha P} - 1\right]$. Therefore $(-1)^k \frac{d^k g(P)}{dP^k} = \alpha^{k-1}\overline{P}e^{-\alpha P} > 0$.

□

A.2 Proof of theorem 1

Before the proof, let me introduce some useful notation. Each player $i \in \mathcal{N} = \{1, \dots, m\}$, observes prices of some players. Let the set of these players be $\mathcal{O}_i = \{j : a_{ji} = 1\} \subset \mathcal{N}$ (possibly empty set) and vector of these prices $\boldsymbol{p}_i = (p_j)_{j \in \mathcal{O}_i}$. Player i's strategy is $p_i^*(\boldsymbol{p}_i)$. Player i also influences some players, let the set of these players be $\mathcal{I}_i = \{j : a_{ij} = 1\} \subset \mathcal{N}$ (again, possibly empty). Each such player $j \in \mathcal{I}_i$ uses the equilibrium strategy $p_j^*(\boldsymbol{p}_j)$. By definition, $i \in \mathcal{O}_j$, i.e., p_i is one of the inputs in $\boldsymbol{p}_j$. However, i does not necessarily observe all prices in $\boldsymbol{p}_j$, therefore it must make an equilibrium conjecture about these values. Let $p_j^i(p_i, \boldsymbol{p}_i)$ denote player j's action as seen by player i. That is, $p_j^i(p_i, \boldsymbol{p}_i) = p_j^*(\boldsymbol{p}'_j)$, where $\boldsymbol{p}_j = (p'_k)_{k \in \mathcal{O}_j}$ is such that $p'_k = p_k$ if $k \in \mathcal{O}_i$ or $k = i$ and $p'_k = p_k^i(p_i, \boldsymbol{p}_i)$ otherwise. The last step makes the definition recursive, but it is well-defined, as each such step strictly reduces the number of arguments in the function. Finally, there are also some players whose prices that i neither observes nor influences, let this set be $\mathcal{U}_j = \{j : a_{ji} = a_{ij} = 0\} \subset \mathcal{N}$. For these players, i expects the actions to be $p_j^i(\boldsymbol{p}_i)$ defined in the same way as above, but its arguments do not include p_i.

Using this notation, a firm i that observes $\boldsymbol{p}_i$ and sets its price to p_i, expects the price of the final good to be

$$P^i(p_i|\boldsymbol{p}_i) = c_0 + p_i + \sum_{j \in \mathcal{O}_i} p_j + \sum_{j \in \mathcal{I}_i} p_j^i(p_i, \boldsymbol{p}_i) + \sum_{j \in \mathcal{U}_i} p_j^i(\boldsymbol{p}_i). \tag{22}$$

The main idea in the proof is the following. Instead of choosing price p_i to maximize profit $(p_i - c_i)D(P^i(p_i|\boldsymbol{p}_i))$, we can think of player i choosing the final good price P to induce. For this, let me assume that in the relevant range, $P^i(p_i|\boldsymbol{p}_i)$ is smooth and strictly increasing in p_i, so that it has a differentiable and strictly increasing inverse function $f_i(P|\boldsymbol{p}_i)$ such that $P^i(f_i(P|\boldsymbol{p}_i)|\boldsymbol{p}_i) = P$. Then the maximization problem is

$$\max_P [f_i(P|\boldsymbol{p}_i) - c_i]D(P),$$

which leads to first-order condition $f_i'(P|\boldsymbol{p}_i)D(P)+[f_i(P|\boldsymbol{p}_i)-c_i]D'(P)=0$ or equivalently

$$f_i(P|\boldsymbol{p}_i)-c_i=g(P)f_i'(P|\boldsymbol{p}_i). \tag{23}$$

Note that there is one-to-one mapping between representing equilibrium behavior in terms of functions $f_i(P|\boldsymbol{p}_i)$ and in terms of $p_i^*(\boldsymbol{p}_i)$.

Proof. Observe that the equilibrium must be interior, i.e., each $c_i<p_i<\overline{P}$ for each firm. If this is not the case for the firm i, then its equilibrium profit is non-positive. This could be for one of two reasons. First, the equilibrium price of the final good is so high that $D(P)=0$. In this case, all equilibrium profits are non-positive and there must be at least one firm i who, by reducing its price (and anticipating the responses of firms influenced), can make the final good price low enough so that it ensures a strictly positive profit. This would be a profitable deviation. Second, if $P<\overline{P}$ and $p_i\leq c_i$, then firm i can raise its price slightly and increase its profit.

I will first derive necessary conditions for an interior equilibrium and combine them into one necessary condition, which gives equation (9). I then show that it has a unique solution and finally verify that it is indeed an equilibrium by verifying that each firm indeed chooses a price that maximizes its profit.

Let us start with any player i who does not influence any other players, i.e., $\boldsymbol{e}_i'\boldsymbol{A}\mathbf{1}=0$ or equivalently $\mathcal{I}_i=\varnothing$. Then we can rewrite equation (22) as

$$P=c_0+f_i(P|\boldsymbol{p}_i)+\sum_{j\in\mathcal{O}_i}p_j+\sum_{j\in\mathcal{U}_i}p_j^i(\boldsymbol{p}_i). \tag{24}$$

Differentiating this expression with respect to P shows that $f_i'(P|\boldsymbol{p}_i)=1$ (that is, player i can raise the price of the final good by ε by raising its own price by ε). Therefore equation (23) implies $f_i(P|\boldsymbol{p}_i)=c_i+g(P)$. Note that this expression is independent of $\boldsymbol{p}_i$, so I can drop it as an argument for f_i and write simply as $f_i(P)=c_i+g(P)$.

Let us take now any player i and suppose that the optimal behavior of all players $j\in\mathcal{I}_i$ is described corresponding functions $f_j(P)$ that do not depend on the remaining arguments $\boldsymbol{p}_j$. Then we can rewrite equation (22) as

$$P=c_0+f_i(P|\boldsymbol{p}_i)+\sum_{j\in\mathcal{O}_i}p_j+\sum_{j\in\mathcal{I}_i}f_j(P)+\sum_{j\in\mathcal{U}_i}p_j^i(\boldsymbol{p}_i). \tag{25}$$

Differentiating this expression and inserting it to equation (23) gives

$$f_i'(P|\boldsymbol{p}_i)=1-\sum_{j\in\mathcal{I}_i}f_j'(P)\quad\Rightarrow\quad f_i(P|\boldsymbol{p}_i)=g(P)\left[1-\sum_{j\in\mathcal{I}_i}f_j'(P)\right]. \tag{26}$$

This expression is again independent of the arguments $\boldsymbol{p}_i$, which we can therefore drop. Moreover, these arguments give precise analytic expressions for $f_i(P)$ functions. We already saw that $f_i(P)=g(P)=\sum_{k=1}^m\boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P)$ when $\boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}=0$ for all $k>1$ (i.e., players who do not influence anybody). Suppose that every player $j\in\mathcal{I}_i$ has

$$f_j(P)-c_j=\sum_{k=1}^m\boldsymbol{e}_j'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P). \tag{27}$$

Then for player i we must have

$$f_i(P) - c_i = g(P)\left[1 - \sum_{j\in\mathcal{I}_i} f_j'(P)\right] = \underbrace{g(P)}_{=\boldsymbol{e}_i'\boldsymbol{A}^0\mathbf{1}g_1(P)} + \sum_{k=1}^{m}\underbrace{[-g_k'(P)g(P)]}_{g_{k+1}(P)}, \tag{28}$$

which, after change of variables from k to $k-1$ and combining the terms, gives the same expression as in equation (27).[26]

Therefore on-path, when the equilibrium price of the final good is P^*, the individual prices are indeed given by the expressions in the theorem. The price of the final good must be sum of all the input prices, therefore P^* must satisfy

$$P^* = c_0 + \sum_{i\in\mathcal{N}} f_i(P^*) = \underbrace{c_0 + \sum_{i\in\mathcal{N}} c_j}_{=C} + \sum_{k=1}^{m}\underbrace{\sum_{i\in\mathcal{N}} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}}_{=\mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}}\, g_k(P^*),$$

which gives the equation (9).

Below I prove two technical lemmas (lemmas 2 and 3) provide monotonicity properties that imply existence and uniqueness of equilibria. We can rewrite equation (9) as $f(P) = P - C - \sum_{k=1}^m \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P) = 0$. At $P = 0$ we have $f(0) = -C - \sum_{k=1}^m \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(0) < 0$ and $\lim_{P\to\overline{P}} f(P) > 0$. By lemma 3, function $f(P)$ is strictly increasing and therefore $f(P) = 0$ has a unique solution, which is the equilibrium price of the final good $P^* \in (0, \overline{P})$.

Next, in the argument above, we assumed that the inverse function of $f_i(P)$ function is strictly increasing. The construction implied a necessary condition that $f_i(P)$ must satisfy and lemma 3 shows that it implies that $f_i(P)$ is indeed strictly increasing, therefore the inverse function $P^i(p_i|\boldsymbol{p}_i)$ is indeed a well-defined strictly increasing function. Finally, to verify that the solution we found is indeed an equilibrium, we need to verify that the solution we derived is indeed a global maximizer for each firm. Notice that by lemma 3, the optimality condition equation (23) has a unique solution for each firm. Therefore we have identified a unique local optimum for each firm. As we already verified that corner solutions would give non-positive profits for each firm and the interior solution gives strictly positive profit, this must be a global maximizer.

Lemma 2 (Monotonicity of $g_k(P)$)**.** *$g_k(P)$ is $(d(\boldsymbol{A}) + 1 - k)$-times monotone.*

Proof. $g_1(P) = g(P) = -\frac{D(P)}{D'(P)}$ is $d(\boldsymbol{A})$-times monotone by assumption 3. Therefore $g'(P)$ is $(d(\boldsymbol{A}) - 1)$-times and $g_2(P) = -g_1'(P)g(P)$ is $(d(\boldsymbol{A}) - 1)$-times monotone. The rest follows by induction in the same way, if $g_k(P)$ is $(d(\boldsymbol{A}) + 1 - k)$-times monotone, then $g_{k+1}(P) = -g_k'(P)g(P)$ is $(d(\boldsymbol{A}) - k)$-times monotone. □

Lemma 3 (Monotonicity of $f(P), f_i(P)$)**.** *The following monotonicity properties hold*

1. $f(P) = P - C - \sum_{k=1}^m \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P)$ *is strictly increasing,*
2. $f_i(P) = c_i - \sum_{k=1}^m \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P)$ *is strictly increasing for each* $i \in \{1, \ldots, m\}$,

[26]Note that no player can have level-m influences, i.e., $\boldsymbol{e}_i'\boldsymbol{A}^m\mathbf{1} = 0$.

3. $f_i'(P)g(P) = \sum_{k=1}^m \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P)$ *is (weakly) decreasing for each* $i \in \{1, \ldots, m\}$.

Proof. Each $\mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}$ and $\boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}$ is a non-negative integer and each $g_k(P)$ weakly decreasing in $-P$ by lemma 2, which implies weak monotonicity of $f_i'(P)g(P)$. Moreover, when $k = 1$, then $g_1(P) = g(P)$ which is strictly decreasing by assumption 3 and $\boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1} = 1 > 0$, which implies that $f_i(P)$ is strictly increasing. As $P - C$ is strictly increasing, then $f(P)$ is also strictly increasing. □

□

A.3 Proof of lemma 1

Remark: The equilibrium prices in our model, denoted as P^* and p_i^*, are determined by all parameters of the model. In particular, the equilbrium prices depend on the number of monopolists (m), their costs (c_i), and the network's structure ($\boldsymbol{A}$).

Proof. Using the facts that $g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right] > \frac{1}{\alpha}$ and $g_k(P) > 0$ for all $k > 0$, equation (9) gives $P^* = C + \sum_{k=1}^m \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) > C + mg(P^*) > C + \frac{m}{\alpha}$. Similarly for individual prices, $p_i^* = c_i + \sum_{k=1}^m \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) > c_i + g(P^*) > c_i + \frac{1}{\alpha}$.

Using the lower bound for P^*, we can bound $e^{-\alpha P^*} < e^{-\alpha C - \alpha\frac{m}{\alpha}} = e^{-\alpha C}e^{-m}$. Therefore $e^{-\alpha P^*} = O(e^{-m})$. I use this result to prove lemma 4 that shows that $g_1(P^*) = \frac{1}{\alpha} + O(e^{-m})$ and $g_k(P^*) = O(e^{-m})$ for all $k > 1$. Using this, we can define $G^m(P^*)$ as follows

$$G^m(P^*) \equiv \max\left\{g_1(P^*) - \frac{1}{\alpha}, g_2(P^*), \ldots, g_m(P^*)\right\} = O(e^{-m}).$$

Therefore equation (9) gives

$$P^* \leq C + \frac{m}{\alpha} + \sum_{k=1}^m \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}G^m(P^*) = C + \frac{m}{\alpha} + O(e^{-m})B(\boldsymbol{A}), \tag{29}$$

where $B(\boldsymbol{A}) = \sum_{k=1}^m \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}$. Now, note that $B(\boldsymbol{A})$ increases each time an edge is added to $\boldsymbol{A}$, so its upper bound is when the network is most connected (fully sequential decisions) and lower bound with least connected network (simultaneous decisions), so that $m \leq B(\boldsymbol{A}) \leq 2^m - 1$. Therefore $B(\boldsymbol{A}) = O(2^m)$. Inserting this observation to previous expression gives $P^* = C + \frac{m}{\alpha} + O\left(\left[\frac{2}{e}\right]^m\right)$. Finally, for the equilibrium expression for individual prices is

$$p_i^* = c_i + \sum_{k=1}^m \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) = c_i + \frac{1}{\alpha} + O(e^{-m})B_i(\boldsymbol{A}), \tag{30}$$

where $B_i(\boldsymbol{A}) = \sum_{k=1}^m \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}$, which is by the same arguments as above $B_k(\boldsymbol{A}) = O(2^m)$ and therefore $p_i = c_i + \frac{1}{\alpha} + O\left(\left[\frac{2}{e}\right]^m\right)$. □

Lemma 4. *With logit demand* $D(P) = d\frac{e^{-\alpha}}{1+e^{-\alpha P}}$*, functions* $g_k(P)$ *and their derivatives have the following limit properties at* $P = P^*$

1. $g_1(P^*) = \frac{1}{\alpha} + O(e^{-m}) = O(1)$*,* $g_k(P^*) = O(e^{-m})$ *for all* $k \in \{2, \ldots, m\}$*,*

2. $\frac{d^l g_k(P^*)}{dP^l} = O(e^{-m})$ *for all* $k, l \in \{1, \ldots, m\}$.

Proof. We showed that $e^{-\alpha P^*} = O(e^{-m})$. Consider $g_1(P^*)$ first. We get $g_1(P^*) = g(P^*) = \frac{1}{\alpha} + \frac{1}{\alpha} e^{-\alpha P^*} = \frac{1}{\alpha} + O(e^{-m}) = O(1)$. Therefore, $\frac{d^l g_1(P^*)}{dP^l} = -(-\alpha)^{l-1} e^{-\alpha P^*} = O(e^{-m})$.

The rest of the proof is by induction. Suppose that the claim holds for $g_1, \ldots, g_k$. Now, $g_{k+1}(P^*) = -g'_k(P^*)g(P^*) = O(e^{-m})$ as $g(P^*) = O(1)$ and $g'_k(P^*) = O(e^{-m})$ by induction assumption. Each derivative can be written as

$$\frac{d^l g_{k+1}(P^*)}{dP^l} = -\sum_{j=0}^{l} \binom{l}{j} g_k^{(l-j+1)}(P^*) g^{(j)}(P^*) \tag{31}$$

Each $g_k^{(l-j+1)}(P^*) = O(e^{-m})$ by induction assumption (as $l-j+1 \geq 1$). When $j = 0$, then $g^{(j)}(P^*) = g(P^*) = O(1)$. Therefore the first element of the sum is $g_k^{(l-0+1)}(P^*)g^{(0)}(P^*) = O(e^{-m})$. For all other elements $j > 0$, so the term $g^{(j)}(P^*) = O(e^{-m})$ and therefore each $g_k^{(l-j+1)}(P^*)g^{(j)}(P^*) = O(e^{-2m})$. This proves that $\frac{d^l g_{k+1}(P^*)}{dP^l} = O(e^{-m})$. □

B Examples of Payoff Functions that Violate Assumption 3

While there are many natural demand functions that satisfy assumption 3, some do not. In this appendix, I discuss examples of such demand functions, i.e., functions $D(P)$ for which the function $g(P) = -D(P)/D'(P)$ is not $d(\boldsymbol{A})$-times monotone in P.[27] Note that even in this case, (9) in theorem 1 remains a necessary condition for an interior equilibrium, but not a sufficient one. As we will see, without assumption 3, a pure-strategy equilibrium may not exist or be unique, and the qualitative properties derived in the paper might not hold. However, the characterization remains instructive, making it easier to find candidates for equilibria and understand the economics of these failures.

B.1 Isoelastic Demand

Suppose that $D(P) = P^{-\varepsilon}$, where $\varepsilon > 1$ represents the demand elasticity. In this case, $g(P) = -\frac{D(P)}{D'(P)} = \frac{P}{\varepsilon}$, which is not strictly decreasing, so it does not satisfy assumption 3. In fact, the condition $\varepsilon > 1$ is required because, otherwise, even assumption 2 is not satisfied.[28]

If there are m firms making simultaneous choices, the necessary condition (9) simplifies to

$$P - C = m g_1(P) = m\frac{P}{\varepsilon},$$

or equivalently, $P^* = \frac{\varepsilon C}{\varepsilon - m}$. For this to be a valid equilibrium price, we need to have $C > 0$ and $\varepsilon > m$. In fact, these conditions are sufficient for the uniqueness and existence of an interior equilibrium in this case. Note that we need a stronger parametric restriction,

[27] I am grateful to an anonymous referee for suggesting this discussion and these functional forms.

[28] Then $PD(P) = P^{1-\varepsilon}$ is increasing, indicating that there is not even an interior optimum for the standard monopoly problem. This is a manifestation of a textbook result, where, in a monopoly optimum, the demand elasticity must be above one.

which is natural, as the functional form of the $D(P)$ function does not guarantee the concavity of profit functions automatically; a sufficiently high elasticity is required. However, once these conditions are satisfied, we do have the multiple-marginalization problem as before, since P^* is increasing in m.

Suppose now that we have m monopolists, and $m_1 > 0$ are the first-movers who make their choices in the first period simultaneously, while the remaining $m_2 = m - m_1 > 0$ observe their choices and move in the second period simultaneously. Then, equation (9) becomes

$$P - C = mg_1(P) + m_1 g_2(P) = m\frac{P}{\varepsilon} - m_1\frac{P}{\varepsilon^2} \quad \Rightarrow \quad P^* = \frac{\varepsilon C}{\varepsilon - m + \frac{m_1}{\varepsilon}}.$$

Note that now the condition $\varepsilon > m$ can be relaxed slightly due to the positive term $\frac{m_1}{\varepsilon}$. This effect comes from the fact that $g_2(P) = -\frac{P}{\varepsilon^2} < 0$, which means that actions are strategic complements. Therefore, the m_1 leaders have an incentive to reduce their prices to encourage the followers to lower their prices as well, so the requirement for price elasticity is not as high anymore.

For example, if $m = 3$ and $m_1 = 2$, so that $m_2 = 1$, then a sufficient condition for the existence of equilibrium is now $\varepsilon > 2$, whereas in the three-player simultaneous case, we required a stronger condition $\varepsilon > 3$. The multiple-marginalization is now decreased with strategic impacts by the m_1 players, as P^* is clearly decreasing in m_1. Again, this comes from the fact that prices are direct strategic complements here, whereas if assumption 3 were satisfied, they would be strategic substitutes.

We can proceed in the same way to study more complex networks. For example, consider focusing again on three players and let's make the game fully sequential. Then, the equation equation (9) becomes:

$$P - C = 3g_1(P) + 3g_2(P) + g_3(P) = 3\frac{P}{\varepsilon} - 3\frac{P}{\varepsilon^2} + \frac{P}{\varepsilon^3} \quad \Rightarrow \quad P^* = \frac{C\varepsilon^3}{(\varepsilon - 1)^3}.$$

We see that now it suffices to have $\varepsilon > 1$, which means that we don't need any extra assumptions compared to a standard monopoly problem. There is still a marginalization problem, as $\frac{C\varepsilon^3}{(\varepsilon-1)^3} > \frac{C\varepsilon}{\varepsilon-1}$. However, the marginalization problem is now lessened compared to networks where monopolists move simultaneously or in two periods. Intuitively, this result balances two effects. On the one hand, $g_3(P) = \frac{P}{\varepsilon^3} > 0$, so the next higher-order effect points towards strategic substitutes again. In other words, influences through influences lead to discouragement. However, compared to simultaneous moves or moves in only two periods, we also added more direct influences, which had the opposite sign. In this functional form, the direct effect dominates the indirect one.

B.2 Mixtures of Regular Demand Functions

Another functional violation of assumption 3 occurs when we use a mixture of two standard demand functions, such as $D(P) = \lambda(1-P)^2 + (1-\lambda)(1-P)^{1/2}$ for $\lambda \in (0, 1)$. In the limits where $\lambda = 0$ or $\lambda = 1$, we have power functions $D(P) = (1-P)^{1/a}$ with $a = 1/2$ and $a = 2$ respectively. Both satisfy assumption 3. In fact, these are simple cases where $g(P) = a(1-P)$ is linear and thus $g_k(P) = a^k(1-P)$ functions are linear, making the calculations especially simple.

However, for $\lambda \in (0,1)$, these convenient properties fail. For example, with $\lambda = \frac{1}{2}$:

$$g(P) = -\frac{D(P)}{D'(P)} = \frac{(1-P)^2 + (1-P)^{1/2}}{2(1-P) + \frac{1}{2}(1-P)^{-1/2}}.$$

This is a non-linear function, which turns out to be positive and strictly decreasing in P. Therefore, in the case of simultaneous decisions, theorem 1 still holds. When we now compute:

$$g_2(P) = -g'(P)g(P) = \frac{2\left(P^2 - 2P + \sqrt{1-P} + 1\right)}{\left(4P\sqrt{1-P} - 4\sqrt{1-P} - 1\right)^3} \times \left(-8P^2(1-P)^{3/2} - P^2 + 16P(1-P)^{3/2} - 8(1-P)^{3/2} + 2P - 2\sqrt{1-P} - 1\right),$$

which is a positive but non-monotone function. Therefore, assumption 3 is not satisfied, although actions are strategic substitutes. Thus, whether the solution to the necessary condition in theorem 1 is indeed an equilibrium requires additional verification. When we proceed by computing $g_3(P), g_4(P)$, and so on, the expressions become too long to display here. They are highly non-linear, non-monotone, and take both positive and negative values.

An alternative aggregation. The calculations above point to an alternative way to aggregate the demand function. The spirit of this example is to consider a demand function "between" two basic demand functions, and the meaning of betweenness was a weighted average. It turns out that this weighted average does not inherit the nice properties of the original demand functions because the demand function itself was not the right object to work with. A more relevant function is the corresponding $g(P)$ function.

Consider k demand functions $D^1, \ldots, D^k$ that all satisfy assumption 3 and weights $\lambda_1, \ldots, \lambda_k > 0$ such that $\sum_{j=1}^k \lambda_j = 1$. Let $g^j(P) = -D^j(P)/\frac{dD^j(P)}{dP}$. Now define

$$g(P) = \sum_{j=1}^k \lambda_j g^j(P)$$

and the corresponding demand function $D(P) = e^{-\int 1/(g(P)dP}$. Then by construction, the new demand function $D(P)$ also satisfies assumption 3.

For example, let $D^j(P) = (1-P)^{1/a_j}$ for $j \in 1, \ldots, k$. Then $g^j(P) = a_j(1-P)$, so that $g(P) = \sum_{j=1}^k g^j(P) = a(1-P)$, where $a = \sum_{j=1}^k \lambda_j a_j$. This function $g(P) = a(1-P)$ corresponds to the demand function $D(P) = (1-P)^{1/a} = (1-P)^{1/\sum_{j=1}^k \lambda_j a_j}$.

Even more specifically, if we take the same parameters as above, i.e., $k = 2, a_1 = 1/2, a_2 = 2$ and $\lambda_1 = \lambda_2 = 1/2$, then we get $a = 5/4$ and therefore the "intermediate" demand function between $(1-P)^2$ and $(1-P)^{1/2}$ would be $(1-P)^{5/4}$. This function inherits all the relevant properties of the two original demand functions and could be easily analyzed by the tools developed in the paper.

B.3 Tullock Contest

Finally, consider a demand function $D(P) = \frac{v}{P} - c$ and $c_i = 0$ for all firms. In this case,

$$\pi_i = (p_i - 0)\left(\frac{v}{P} - c\right) = \frac{p_i}{P}v - cp_i,$$

This is a payoff function of a (Tullock) contest where firms choose costly actions at a marginal cost c to win a prize of value v, with the probability of winning being p_i/P. This payoff function does not satisfy assumption 3, because $g(P) = P\left(1 - \frac{c}{v}P\right)$ is not a monotone function.

Nevertheless, Hinnosaar (2023) proves that at least in a special case of a sequential contest, the sufficient conditions for existence and uniqueness are still satisfied. That is, when players can be divided into T groups, where players in each group observe the actions of all players in previous groups (or at least the sum of these actions) and move simultaneously with players in the same group. Moreover, Hinnosaar (2023) also shows that each $g_k(P) \geq 0$ for P close to the equilibrium value P^*. Therefore, the implications on multiple-marginalization and influentiality still hold.