Price Setting on a Network*

Toomas Hinnosaar†

August 2024‡

Abstract

I study price setting within a network of interconnected monopolists. Some firms possess stronger commitment or bargaining power than others, enabling them to influence the pricing decisions of other firms. While it is well-understood that multiple marginalization reduces both total profits and social welfare, I show that strategic interactions within the network exacerbate the marginalization problem. Individual profits are proportional to a new measure of network centrality, defined by the equilibrium characterization. The results underscore the importance of network structure in policy considerations, such as mergers or trade policies.

JEL: C72, L14, D43
Keywords: price setting, network centrality, sequential games, multiple-marginalization

1 Introduction

Most products are produced and sold through a network of interconnected producers, intermediaries, and retailers. These firms maximize profits, often wielding significant market power. Furthermore, their pricing decisions are interdependent—by charging higher prices, some firms can directly influence the pricing strategies of others. For instance, in the book publishing industry, a publisher sources content, outsources printing, and relies

*I am grateful to Gary Biglaiser, Federico Boffa, Doruk Cetemen, Eddie Dekel, Matt Elliott, Jeff Ely, Francesco Giovannoni, Ben Golub, Gautam Gowrisankaran, Sanjeev Goyal, Edoardo Grillo, Andrei Hagiu, Marit Hinnosaar, Matt Jackson, Maciej Kotowski, Rachel Kranton, Mihai Manea, Alexander Matros, Konrad Mierendorff, Ignacio Monzón, Martin Peitz, Kyle Phong, Salvatore Piccolo, Fernando Vega-Redondo, David Rivers, Bill Rogerson, József Sákovics, Marco Serena, Georg Schaur, Nicolas Schutz, Cristián Troncoso-Valverde, Asher Wolinsky, Julian Wright, Huseyin Yildirim and seminar participants at the Collegio Carlo Alberto, U. South Carolina, U. Tennessee, Appalachian State U., U. Bath, Cardiff B.S., Bristol, SMU, Duke, UNC, UIUC, Northwestern, U. Houston, U. Alberta, UBC, U. Washington, U. Nottingham, U. Surrey, Lancaster U., Durham U., U. Cambridge, U. Essex, U. Edinburgh, UCL, U. Bonn, Humboldt U, U. Mannheim, U. Tartu, U. Manchester, JEI (Barcelona), Lancaster Game Theory Conference (Lancaster), UniBg Economic Theory Symposium (Bergamo), Barcelona GSE Summer Forum, Contests: Theory and Evidence (East Anglia), SAET 2019 (Ischia), International Conference on Game Theory (Stony Brook), Midwest Economic Theory Meeting (St Louis), SEA (Ft. Lauderdale), CMID20, and Compass Lexecon Economics Conference in Oxford for their comments and suggestions.

†University of Nottingham and CEPR, `toomas@hinnosaar.net`.

‡First draft: May 2018. The latest version: `toomas.hinnosaar.net/pricing_network.pdf`.

on distributors to reach retail chains. Many of these players possess market power and earn positive profits.[1]

This paper addresses the question: How do firms in a network set prices for their products when they have market power and can influence other firms' pricing decisions? The main result is a characterization theorem. Under certain regularity conditions, a unique equilibrium exists. The equilibrium condition has a natural interpretation: it equalizes the difference between the equilibrium price of the final good and the total marginal cost with a weighted sum of *influences* at all levels. At the most basic level, a firm's profit is directly affected by its own price increase. At the next level, a firm's price increase can alter the behavior of directly connected firms. Furthermore, each price change can ripple through the network, influencing indirectly connected firms. These influences are weighted by endogenous factors determined by the demand function's shape and equilibrium behavior.

How do social welfare and total profits depend on network structure? As predicted by existing literature, in this model of firms with market power, multiple marginalization is the primary distortion that reduces both profits and welfare. The key new insight from this analysis is that network structure plays a crucial role in determining the extent of the marginalization issue. I show that strategic interactions within a network magnify the marginalization problem. For example, while a merger might seem to enhance efficiency based on conventional wisdom, if it increases the control of the merged firms, it could potentially negate the expected efficiency gains.

The rest of the paper is structured as follows. The next section discusses the related literature. Section 3 introduces the model and discusses its assumptions. Section 4 provides the main characterization result and describes the main insights of the analysis. Section 5 interprets the characterization by comparing it with some known benchmarks and then discusses the key distortion—the magnification of multiple-marginalization. Section 6 studies which firms are more influential and discusses the relationship between the implied influentiality measure and standard network centrality measures. Section 7 describes how to apply the characterization result to compute the equilibrium and provides further results for some of the most common demand functions. Section 8 concludes. All proofs are in appendix A.

2 Related Literature

Industrial organization. This paper contributes to the literature on vertical integration. Spengler (1950) was the first to describe the double-marginalization problem, and since then, the literature has extensively studied the benefits and costs of vertical control, including Mathewson and Winter (1984), Grossman and Hart (1986), Rey and Tirole (1986), Salinger (1988), Salinger (1989), Riordan (1998), Ordover et al. (1990), Farrell and Shapiro (1990), Bolton and Whinston (1993), Kuhn and Vives (1999), Nocke and White (2007), and Buehler and Gärtner (2013). Empirical work shows that production

[1] In a typical \$26 book, retailers take about 50%, 13% covers printing and transport, and authors receive around 15%, indicating significant markups. Source: New York Times article 'Math of Publishing Meets the E-Book' by Motoko Rich (Feb. 28, 2010), `https://www.nytimes.com/2010/03/01/business/media/01ebooks.html`.

often has a network structure (Atalay et al., 2011), and that mergers or the removal of vertical restraints may sometimes harm consumers (Gayle, 2013; Crawford et al., 2018; Luco and Marshall, 2020). While the theoretical literature analyzes various forms of competition and contract structures, little is known about networks where firms with market power operate at more than two levels (upstream-downstream). In this paper, I focus on monopolistic producers and posted price contracts, extending the analysis to general network structures. I show that strategic interactions on networks magnify multiple-marginalization distortions. This magnification effect is missing from previous literature because it requires accounting for indirect influences, which can only occur on networks with more than two levels of interaction.

Nava (2015) is an exception, studying Cournot competition where trades are restricted by network structures and providing a characterization result. Although the setup differs from mine, it also identifies marginalization as a major source of inefficiency. However, while inefficiency disappears with a large number of firms in their model, my model does not exhibit this feature.

Network games. The paper contributes to the literature on network games, where players take actions on a fixed network and the payoffs depend both on their own and their neighbors' actions. According to a survey by Jackson and Zenou (2015), most works in this literature can be divided into two groups. First, a lot of progress has been made in games with quadratic payoffs (or more generally, payoffs that imply linear best-responses). A seminal paper is Ballester et al. (2006). It found that the equilibrium actions are proportional to Bonacich centrality. Bramoullé and Kranton (2007), Calvó-Armengol et al. (2009), Bramoullé et al. (2014), and Zhou and Chen (2015) study more general variations of this game and find that Bonacich centrality still determines the equilibrium behavior. Bloch and Quérou (2013) and Fainmesser and Galeotti (2016) study pricing of goods with network externalities with quadratic payoffs and find that optimal pricing leads to discounts that are proportional to Bonacich centrality. Bimpikis et al. (2019) study Cournot competition on a bipartite network, where the sellers Cournot-compete in markets which they have access to. They show that when the demands are linear and costs quadratic, the equilibrium behavior is proportional to Bonacich centrality. The second branch of network games studies games with non-quadratic payoffs and is generally able to analyze only qualitative properties of the equilibria rather than provide a full characterization. [2] A seminal paper is Galeotti et al. (2010).

Compared to these works, this paper provides a characterization result for a game on a network with a relatively general payoff structure. The characterization defines a new measure of influentiality, with firms' choices and payoffs proportional to this measure. In special cases where the best-response functions are linear, this measure is proportional to Bonacich centrality. However, since the demand function and equilibrium behavior endogenously define the weights, the measure of influentiality differs from Bonacich centrality for all other demand functions. I provide examples of special cases where it can

[2] An exception is Choi et al. (2017), which studies price competition on networks, where consumers choose the cheapest paths from source to destination and intermediaries set prices, thus making the game a generalization of Bertrand competition. In settings where the players interact on a network randomly, the analysis is more tractable, for example Manea (2011, 2018); Condorelli et al. (2017) who study bargaining on networks.

be equivalent to degree centrality or even independent of the network structure.[3]

Sequential and aggregative games. Methodologically, this paper builds on recent advances in sequential and aggregative games. The model becomes an aggregative game in a special case where firms make independent decisions. Aggregative games were first proposed by Selten (1970), and there has been recent progress in the literature by Jensen (2010), Martimort and Stole (2012), and Acemoglu and Jensen (2013), which has shed new light on questions in industrial organization, as explored by Anderson et al. (2020) and Nocke and Schutz (2018). One classic example of an aggregative game is a contest, and this paper builds on recent work on sequential contests by Kahana and Klunover (2018) and Hinnosaar (2024), extending the methodology to networks and asymmetric costs.[4]

3 Model

3.1 Setup

The model is static and studies the supply of a single final good. The final good has a demand function $D(P)$, where P is its price. The production and supply process requires m inputs in constant proportion. I normalize the units of inputs so that one unit of each input is required to produce one unit of output.

Input i is produced by a monopolistic firm i, that has a constant marginal cost c_i and a price p_i for its product. The price p_i is firm i's per-unit revenue net of payments to other firms in the model. Due to normalization, the quantity of firm i's product (i.e., quantity of input i) is equal to $D(P)$. Therefore firm i gets profit $\pi_i(\boldsymbol{p}) = (p_i - c_i)D(P)$, where $\boldsymbol{p} = (p_1, \dots, p_{m+n})$ and the price of the final good is the sum of all net prices, $P = \sum_{i=1}^{m+n} p_i$. Each monopolist i sets the price p_i strategically, i.e., maximizing profits, anticipating the impact on sales of the final good.

To complete the description of the model, I need to specify how the price p_i of firm i affects the behavior of other monopolists, which I do by introducing the network of influences. Formally, a network of influences consists of all m monopolists as nodes and edges that define influences. The edges are described as an $m \times m$ adjacency matrix $\boldsymbol{A}$, where an element $a_{ij} = 1$ indicates that firm i *influences* firm j. That is, when firm j chooses price p_j, then it takes price p_i as given and responds optimally to it. Of course, firm i knows this and when choosing p_i, it knows that j will respond optimally. Finally, if i and j are not directly linked, i.e., $a_{ij} = a_{ji} = 0$, then neither responds to deviations

[3] Networks also play an important role in international trade and macroeconomics. Trade exhibits a network structure (Chaney, 2014), with early works integrating vertical restraints and intermediaries' roles (Spencer and Jones, 1991; Antràs and Costinot, 2011). Recent literature focuses on network formation in production (Oberfield, 2018; Liu, 2019). Compared to this literature, all firms in my model have market power.

[4] Other papers at the intersection of contests and network literature include Franke and Öztürk (2015), Matros and Rietzke (2018), Kovenock and Roberson (2018), Cortes-Corrales and Gorny (2024), Dziubiński et al. (2021), and Amarasinghe et al. (2023), who study contests on networks, and Goyal et al. (2019), who studies contagion on networks. The approach and questions studied in these works differ from those in my paper.

by the other firm. They expect the other firm to behave according to its equilibrium strategy. For convenience, I assume that the diagonal elements $a_{ii} = 0$. I will discuss a few examples of the network of influences in section 3.4.

In the end if this section, I will discuss the interpretation of the model in more detail. Specifically, section 3.3 discusses the role and the generality of the assumptions about the setup and section 3.4 discusses the interpretation of the network structures. However, before I do this, I make three additional regularity assumptions that help with tractability and guarantee the existence and uniqueness of the equilibrium.

3.2 Regularity Assumptions

The first regularity assumption specifies the class of networks.

Assumption 1. *Network $\boldsymbol{A}$ is acyclic and transitive.*[5]

It is natural to assume that the network is acyclic. If firm j takes p_i as given, then firm i cannot simultaneously take p_j as given. A similar argument applies to cycles involving more than two players. This assumption allows firms i and j to make independent choices when $a_{ij} = a_{ji} = 0$, and the network does not need to be connected.[6]

The transitivity assumption requires that if firm i influences firm j and firm j influences firm k, then firm i also directly influences firm k. In other words, firm k takes both p_j and p_i as given. This assumption was naturally satisfied in the examples discussed earlier. Relaxing the transitivity assumption introduces the possibility of indirect observability, where firms might infer information about the prices of others through the prices they observe. For instance, consider a network with three firms: 1, 2, and 3, where 1 influences 2, and 2 influences 3, but 1 does not have a direct influence on 3. In this case, firm 3 observes p_2 and knows that it is chosen optimally for a given p_1, but does not observe p_1. Thus, firm 3 would need to make an inference about p_1 based on p_2. Of course, firm 2 knows that firm 3 would make such an inference and might try to manipulate it. Such indirect observability would substantially complicate the analysis. The transitivity assumption eliminates the need for indirect observability, thereby simplifying the analysis considerably.

To be clear, both assumptions—acyclicity and transitivity—are restrictive and may limit the model's applicability. They serve different purposes in the analysis. Acyclicity restores a sense of sequentiality to the model, enabling the construction of equilibrium conditions and helping to guarantee equilibrium existence. In networks with cycles, firms can influence each other in a potentially infinite loop, leading to influences of an infinitely high degree. As I show below, with certain demand functions, such an infinite sequence of influences can easily result in the non-existence of equilibrium. Transitivity is primarily a technical assumption that simplifies the model by ruling out indirect observability, which would otherwise add considerable complexity to the analysis.

The second regularity assumption puts standard restrictions on the demand function. The demand function $D(P)$ is a smooth and strictly decreasing function. It either has a

[5]Acyclicity: $\nexists i_1, \ldots, i_k$ such that $a_{i_1 i_2} = \cdots = a_{i_{k-1} i_k} = a_{i_k i_1} = 1$. Equivalently, $\boldsymbol{A}^m = \boldsymbol{0}$, where m is the number of monopolists. Transitivity: if $a_{ij} = a_{jk} = 1$, then $a_{ik} = 1$. Equivalently, $\boldsymbol{A} \geq \boldsymbol{A}^2$.

[6]Acyclicity also helps with tractability. Recent papers by Galeotti et al. (2021) and Pellegrino (2023) explore related models without acyclicity, but they do not allow for sequential decisions.

finite saturation point $\overline{P}$ at which the demand is zero or converges to zero fast enough so that the profit maximization problem is well-defined.

Assumption 2. *Demand function $D : [0, \overline{P}) \to \mathbb{R}_+$ is continuously differentiable and strictly decreasing in $[0, \overline{P})$ where $\overline{P} \in \mathbb{R}_+ \cup \{\infty\}$. Moreover, it satisfies limit condition $\lim_{P \to \overline{P}} PD(P) = 0$.*

The third and final regularity assumption ensures that the demand function $D(P)$ is well-behaved, so local first-order optimality conditions define that equilibrium. It is common in the literature to make a regularity assumption that D is twice differentiable and profits single-peaked. In particular, in theoretical works the demand is often assumed to be linear for tractability. However, in empirical literature logit demand is more common. Here, I make an assumption about the demand function that would be analogous to the standard regularity assumption and covers both linear and logit demand functions.

Let the depth of the network $d(\boldsymbol{A})$ be the length of the longest path in $\boldsymbol{A}$.[7] Moreover, let me define a function

$$g(P) = -\frac{D(P)}{D'(P)}, \tag{1}$$

which is a convenient alternative way to represent the demand function. Note that $g(P) = \frac{P}{\varepsilon(P)}$, where $\varepsilon(P) = -\frac{dD(P)}{dP}\frac{P}{D(P)}$ is the demand elasticity.[8] Then I make the following assumption about the shape of the demand function.

Assumption 3. *$g(P)$ is strictly decreasing and $d(\boldsymbol{A})$-times monotone in $P \in (0, \overline{P})$, i.e., for all $k = 1, \dots, d(\boldsymbol{A})$, derivative $\frac{d^k g(P)}{dP^k}$ exists and $(-1)^k \frac{d^k g(P)}{dP^k} \geq 0$ for all $P \in (0, \overline{P})$.*

To interpret the condition, let us look at the standard monopoly pricing problem $\max_P \pi(P) = \max_P (P - C)D(P)$. Then the first-order necessary condition for optimality of P^* is

$$\pi'(P^*) = D(P^*) + (P^* - C)D'(P^*) = 0 \quad \Longleftrightarrow \quad P^* - C = g(P^*), \tag{2}$$

which illustrates the convenience of the $g(P)$ notation. Moreover, a sufficient condition for optimality is $\pi''(P^*) < 0$ or equivalently $2[D'(P^*)]^2 > D(P^*)D''(P^*)$. Note that a sufficient condition for this is $[D'(P^*)]^2 > D(P^*)D''(P^*)$, which is equivalent to $g'(P^*) < 0$. Therefore, in the standard monopoly problem, the monotonicity of $g(P)$ guarantees that monopoly profit has a unique maximum that can be found using the first-order approach. For general networks, the condition is stronger, as it also guarantees that best-responses and best-responses to best-responses are well-behaved so that the first-order approach is valid.

As illustrated by the monopoly example, the condition is sufficient and not necessary, but it is easy to check, and it is satisfied for many applications. The following proposition provides a formal statement by showing that with many typical functional form assumptions on $D(P)$, the function $g(P)$ is completely monotone, i.e., d-times monotone for arbitrarily large $d \in \mathbb{N}$. Therefore assumption 3 is satisfied with all networks.

[7] Formally, $d(\boldsymbol{A})$ is smallest d is such that $\boldsymbol{A}^d = \boldsymbol{0}$. For instance, in example 3 in the subsection 3.4 (figure 2b) depth $d(\boldsymbol{A}) = 3$, from the path $R \to D \to F$.

[8] Technically, $g(P)$ is the reciprocal of the demand semi-elasticity.

Proposition 1 (Many demand functions imply completely monotone $g(P)$)**.** *Each of the following demand functions implies d-times monotone $g(P)$ for any $d \in \mathbb{N}$:*

1. *Linear demand $D(P) = a - bP$ with $a, b > 0 \Rightarrow g(P) = \overline{P} - P$, where $\overline{P} = \frac{a}{b} > 0$.*
2. *Power demand $D(P) = d\sqrt[\beta]{a - bP}$ with $d, \beta, a, b > 0 \Rightarrow g(P) = \beta(\overline{P} - P)$.*
3. *Logit demand $D(P) = d\frac{e^{-\alpha P}}{1+e^{-\alpha P}}$ with $d, \alpha > 0 \Rightarrow g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right]$.*
4. *Exponential demand $D(P) = a - be^{\alpha P}$ with $a > b > 0, \alpha > 0 \Rightarrow g(P) = \frac{1}{\alpha}\left[\overline{P}e^{-\alpha P} - 1\right]$.*

Note that for all four functions assumption 2 is clearly also satisfied. Linear and power demand functions have saturation point $\overline{P}$, logit demand satisfies $\lim_{P\to\infty} PD(P) = d\lim_{P\to\infty}\frac{1}{\alpha e^{\alpha P}} = 0$, and exponential demand has saturation point $\overline{P} = \frac{1}{\alpha}\log\frac{a}{b}$.[9]

3.3 Discussion of Assumptions

Let me make a few remarks about the model here. First, the network of influences is a *reduced-form* way to capture sequential interactions. I will discuss more examples in the next subsection, but one natural way to interpret it is through the lens of commitment power—some firms may have more commitment power than others in their pricing decisions. An alternative interpretation is bargaining power—they can make take-it-or-leave-it offers. In this paper, I take the network as a fixed primitive of the model and do not explicitly model its microfoundations. The network of influences makes the game sequential. If $a_{ij} = 1$, then firm i sets its price p_i before firm j. Firm j then observes p_i and may respond optimally. Of course, firm i knows this and can anticipate the response of firm j. I am looking for pure-strategy perfect Bayesian equilibria, where players take some of the choices of other players as given and maximize their profits, anticipating the impact on other players' choices and the final good demand.[10]

Second, I make an extreme assumption that a separate monopolist produces each input. It is straightforward to extend the model to situations where some inputs are produced by price-taking firms. This assumption would cover cases where these firms operate in a perfectly competitive sector or compete as Bertrand competitors, in which case the price is equal to the marginal cost of the second cheapest firm in this sector. The firm could also operate in a regulated industry where its price is set by a regulator. In all these cases, such an extension would simply mean that the price of the particular input is equal to a constant, which will be added to the total marginal cost C in the equilibrium characterization.[11] This extension does not require that each firm is either always a price-taker or always a monopolist. The assumption used in the characterization is that monopolists behave according to their local optimality condition, whereas price-takers take their prices locally as given. A firm could be a monopolist in one situation and

[9]Of course, not all demand functions satisfy assumption 3. In the working paper version of the paper, I discuss some such examples. When assumption 3 is violated, there can still be unique equilibrium, but also multiple equilibria or no equilibria at all.

[10]Although I am not excluding the possibility of mixed-strategy equilibria, I show that there always exists a unique pure-strategy equilibrium, so it is natural to focus on it.

[11]See the working paper version of the paper for this extension.

a price-taker when the model parameters change. However, covering cases between these two extremes—where input is produced by an oligopolistic market in which multiple firms have market power—is substantially more complex and will be left for future research.

Third, firms choose net prices p_i. As the marginal costs are known and constant, this is equivalent to assuming that firms choose dollar markups $p_i - c_i$. This assumption is consistent with the empirical fact that the majority of firms in practice employ cost-plus pricing Noble and Gruca (1999), i.e., they add a markup to their costs. It is less clear whether this markup is defined in terms of a percentage or a dollar amount. Most authors model it as a percentage markup, but there is also literature focusing on dollar markups (Jeuland and Shugan, 1988; Choi, 1991; Wang et al., 2013). Wang et al. (2013) shows that even in a simple model, these two assumptions are not equivalent. This equivalence fails when firms are making simultaneous commitments. It is easy to see that in other settings, such as in a fully sequential decision-making process, the two assumptions are equivalent, and they are both equivalent to firms choosing cumulative prices.[12]

Fourth, the assumption that inputs are used in constant proportions (Leontief production function) is clearly restrictive, but it is a relatively common assumption in the literature (Costinot et al., 2013) and helpful in terms of tractability. Relaxing this assumption would be interesting, but its complexity is beyond the scope of this paper.

3.4 Examples of Network of Influences

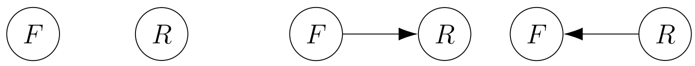
The network of influences I introduce in this paper is motivated by, but certainly not the same as, the supply-chain network. A typical supply-chain network model specifies the flows of goods and services (material flows), as well as the flows of money and information. The specifics of these flows are neither necessary nor sufficient to characterize pricing decisions.[13] For pricing decisions, the model needs to specify what is known to each monopolist when it makes a pricing decision and how it expects this decision to influence the choices of other firms. In other words, the model needs to specify the observability of prices and the commitment power of firms. As described above, I model this by assuming that there is a commonly known network, such that whenever there is an edge from i to j, firm j observes p_i and therefore takes it into account in its optimization problem.

Consider a simple case with just two firms, F (final goods producer) and R (retailer). Then there are three possible networks, illustrated by figure 1. First, figure 1a where firms set their prices p_F and p_R independently, and the final good is sold at $P = p_F + p_R$. This could be a reasonable assumption, for example, if both are large firms that interact with many similar firms. In this case, the final goods producer F does not best-respond to a particular retailer R but to the equilibrium price p_R^* of a representative retailer. Similarly, the retailer does not best-respond to deviations by a particular producer F, but to the equilibrium price p_F^* of a representative producer. Another example where it is natural to

[12] Consider m sequential firms and let the costs be zero. Firm k buys the product from firm $k-1$ and pays $P_{k-1} = \sum_{i=1}^{k-1} p_i$ for it. From firm k's perspective, P_k is fixed, so it is evident that instead of choosing net price (or dollar markup) p_k, the firm's optimization problem could be written as choosing cumulative price $P_k = P_{k-1} + p_k$ or percentage markup $m_k = \frac{p_k}{P_{k-1}}$, as in the optimum, all three choices lead to the same cumulative P_k, which is the only variable affecting the choices of other firms.

[13] Indeed, my model can alternatively be interpreted as a system of perfectly complementary products, where producers make their pricing decisions over time (Matutes and Regibeau, 1992).

make this assumption is when two firms are separately selling perfectly complementary products to final consumers.

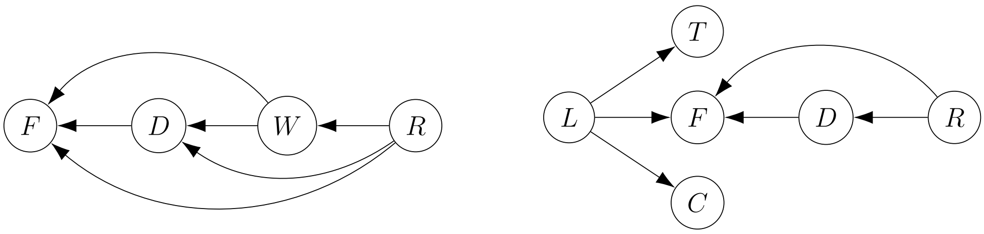


(a) Example 1a (b) Example 1b (c) Example 1c

Figure 1: Example: Three possible two-player networks

Similarly, there could be many reasons for strategic influences. For example, a downstream influence from producer F to retailer R (figure 1b) may arise with a large producer and a small retailer, where the representative retailer reacts optimally to pricing by F. The large producer knows that retailers respond to its pricing and takes into account how a representative retailer best-responds. Conversely, the influence could go in the opposite direction (as in figure 1c) for the same reason—a large retailer R knows that a small producer F will best-respond to its price changes. In this paper, I take these influences as given and simply assume that some firms have more commitment power than others for exogenous reasons.

Let me illustrate the network of influences with two more examples. Figure 2a depicts an example of a retail chain with downstream-to-upstream influences. In this example, there is a strong retailer R, who can commit to adding a markup p_R on top of the wholesale price P_W, so that the price of the final good will be $P = P_W + p_R$. The wholesaler W takes p_R as given and commits to its markup p_W, so that when the distributor's price is P_D, the wholesale price is $P_W = P_D + p_W$, and therefore the final good price is $P = P_D + p_W + p_R$. Then distributor D sets its markup p_D, taking markups p_W and p_R as given. Finally, the final good producer F sets a price p_F, taking into account that the final consumer will pay $P = p_F + p_D + p_W + p_R$.



(a) Example 2 (A retail chain with upstream-to-downstream influences)

(b) Example 3 (A network with small producer and a common raw-material producer)

Figure 2: Two examples of networks with more than two firms

As argued above, influences can also go in the opposite direction—from downstream to upstream. Moreover, there is no reason to assume that the flows of influence all move in the same direction or that the network is a tree. Figure 2b provides another example, where the same raw material L (labor) is used by three firms: T (transport), F (final goods producer), and C (communication). These three firms set their prices independently, but F additionally takes the markups of the D (distributor) and R (retailer) as given.

4 Characterization

In this section, I first discuss two examples to illustrate the equilibrium characterization. The main result of the paper is the characterization theorem that formalizes the approach.

4.1 Example: Non-linear Demand

The first example illustrates the complexities that arise from working with a non-linear demand function when the decisions are sequential. Let us consider logit demand $D(P) = \frac{e^{-P}}{1+e^{-P}}$, costless production, and two monopolists who choose their prices sequentially, i.e. as in example 1b in figure 1b. That is, first firm 1 chooses price p_1, which is taken into account by firm 2 when it chooses price p_2. The price of the final good is $P = p_1 + p_2$.

The standard method of finding the equilibrium in this game is backward induction. It starts by finding the best-response function of firm 2, by solving $\max_{p_2} p_2 D(p_1 + p_2)$. The optimality condition is

$$\frac{d\pi_2}{dp_2} = D(p_1 + p_2) + p_2 D'(p_1 + p_2) = 0 \iff e^{p_2}(1 - p_2) = e^{-p_1}. \tag{3}$$

Solving this gives the best-response function $p_2^*(p_1) = 1 + W(e^{-(p_1+1)})$, where $W(\cdot)$ is the Lambert W function[14]. Substituting $p_2^*(p_1)$ into the optimization problem of firm 1 gives a maximization problem $\max_{p_1} p_1 D(p_1 + p_2^*(p_1))$. From this, we get the optimality condition

$$1 + e^{-(p_1+1)-W(e^{-(p_1+1)})} = p_1 \left(1 + \frac{-e^{-(p_1+1)} W(e^{-(p_1+1)})}{e^{-(p_1+1)} + W(e^{-(p_1+1)})}\right). \tag{4}$$

Solving the equation numerically gives $p_1^* \approx 1.2088$, therefore $p_2^* \approx 1.0994$ and $P^* \approx 2.3082$. However, optimality condition (4) cannot be solved analytically. This implies that the standard approach is intractable when the network is more complex than the one studied here. Backward induction cannot be used because computing best-response functions and substituting them into the maximization problems of other firms is not feasible. The issue is tractability—as the optimality conditions are non-linear, solving them leads to complex expressions. Replacing best-responses sequentially amplifies these complexities.

The solution to this problem comes from Hinnosaar (2024), which proposed characterizing the behavior of the following players by inverted best-response functions.[15] The key observation is that although equation (3) is a highly non-linear function of p_1 and p_2 separately, fixing $P = p_1 + p_2$ leads to a linear equation for p_2, or equivalently $p_1 = P - p_2$. Therefore, for a fixed price of the final good, P, it is straightforward to find the price of firm 2 that is consistent with the final good price P. I denote this by $f_2(P)$, i.e.,

$$f_2(P) = p_2 = -\frac{D(P)}{D'(P)} = g(P)$$

[14] The function $W(x)$ is defined as a solution to $x = W(x)e^{W(x)}$ (Euler, 1783). For a detailed description, see `https://en.wikipedia.org/wiki/Lambert_W_function`.

[15] Kahana and Klunover (2018) developed independently and concurrently a similar approach to find equilibria in n-player sequential Tullock contests.

where $g(P) = 1 + e^{-P}$. Firm 1 knows that if it sets its price to p_1, the price of the final good will satisfy $P = p_1 + f_2(P)$. Therefore, we can think of firm 1's problem as choosing P to solve $\max_P [P - f_2(P)]D(P)$. Taking the first-order condition gives us

$$f_1(P) = p_1 = P - f_2(P) = g(P)[1 - f_2'(P)].$$

Therefore, if the final good's price in equilibrium is P^*, then the optimal behavior of both players requires that $P^* = f_1(P^*) + f_2(P^*) = 2g(P^*) - g'(P^*)g(P^*)$. This equation is straightforward to solve, and the same argument can be easily extended to more players choosing sequentially.

There is one more pattern in these expressions that the characterization will exploit. Namely, the condition for equilibrium is

$$P^* = 2g(P^*) - g'(P^*)g(P^*) = 2g_1(P^*) + g_2(P^*),$$

where $g_1(P^*) = g(P^*)$ and $g_2(P^*) = -g_1'(P^*)g(P^*)$. The expression on the right-hand side consists of two elements. The first, $2g_1(P^*)$, captures the fact that two players each individually maximize their profits. The second, $g_2(P^*)$, captures the fact that player 1 influences player 2. It is straightforward to verify that, for example, if we were to remove this influence—i.e., with two monopolists choosing their prices simultaneously—the equilibrium condition would become $P^* = 2g_1(P^*)$.

The main advantage of this approach is tractability. Instead of solving non-linear equations at each step and inserting the resulting expressions into the next maximization problems, which leads to increasingly complex non-linear expressions, this approach allows combining all necessary conditions of optimality into one necessary condition. Under assumptions 2 and 3, the resulting expression has a unique solution, which gives us a unique candidate for an interior equilibrium. Under the same assumptions, the sufficient conditions for optimality are also satisfied, thereby determining a unique equilibrium.

4.2 Example: Interconnected Decisions

The second example illustrates a new issue that arises in the case of networks—the decisions are interconnected. In example 3 discussed earlier (figure 2b), firms L and D make independent decisions, but due to their positions, they have different views on what happens before and after them. Firm D influences only F, but L influences T and C as well. Similarly, D takes p_R as given, whereas L does not observe p_R and therefore must form an equilibrium conjecture about the optimal behavior of R. As a result, solving the game using backward induction is no longer possible even if the demand function would be linear.

To illustrate this issue, consider the simple network shown in figure 3. Let us assume that the demand is linear $D(P) = 1 - P$ and all costs are zero. The strategies of the four firms are, respectively, p_1^*, p_2^*, $p_3^*(p_1)$, and $p_4^*(p_1, p_2)$. To find the equilibrium using backward induction, we would first need to start with firms 3 and 4. Firm 4 maximizes profit by taking p_1 and p_2 as given and assuming equilibrium behavior from firm 3. Thus, its first-order condition would be a function of $p_3^*(p_1)$, so we would not be able to compute firm 4's best-response function without solving firm 3's problem. Now, firm 3 maximizes its profit, taking p_1 as given. Its first-order condition would be a function of $p_4^*(p_1, p_2^*(p_1))$

and $p_2^*(p_1)$. Thus, we are not able to solve for firm 3's best-response function before solving firm 2's problem, and so on.

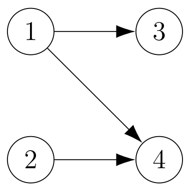


Figure 3: Example 4: network with interconnected decisions

Appendix B performs these somewhat tedious calculations first directly, as sketched out above, and then using the inverted best-response approach. Of course, in both cases, the equilibrium condition boils down to the same condition

$$P^* = 4g_1(P^*) + 3g_2(P^*) = 6(1 - P^*),$$

where $g_1(P) = -D(P)/D'(P) = 1 - P$ and $g_2(P) = -g_1'(P)g_1(P) = 1 - P$. Solving this equation is straightforward, and it gives $P^* = \frac{7}{8}$.

The advantage of the inverted best-response approach is that it combines all necessary conditions into one, avoiding the tedious iterative process of solving the game backwards, then simultaneously, then backwards again, and so on. In other words, the issues of interconnected decisions are automatically mitigated.

4.3 Characterization

As illustrated by the examples above, it is useful to define functions $g_1, \ldots, g_n$, which capture relevant curvature properties of the demand function. They are defined recursively as

$$g_1(P) = g(P) = -\frac{D(P)}{D'(P)} \text{ and } g_{k+1}(P) = -g_k'(P)g(P). \tag{5}$$

As the discussion about monopoly profit maximization and the examples illustrated, $g_1(P)$ captures the standard concavity of the profit function, whereas $g_2(P)$ captures the direct discouragement effect when a firm observes the price of another firm. Functions $g_3, \ldots, g_n$ play a similar role in describing higher-order discouragement effects.

The adjacency matrix $\boldsymbol{A}$ provides a convenient way to keep track of the number of direct and indirect influences. Multiplying the adjacency matrix with a column vector of ones, $\boldsymbol{A1}$, gives a vector with the number of edges going out from each player (i.e., the sum over columns). Similarly, $\boldsymbol{1}'\boldsymbol{A1}$ is the total number of edges in the network, i.e., the total number of direct influences. Multiplying the adjacency matrix by itself, i.e., $\boldsymbol{A}^2 = \boldsymbol{AA}$, gives a matrix that describes two-edge paths, i.e., element $a_{i,j}^2$ is the number of paths from i to j with one intermediate step. Similarly, $\boldsymbol{A}^k$ is the matrix that describes the number of all k-step paths from each i to each j. When we take $k = 0$, then $\boldsymbol{A}^0$ is an identity matrix, which can be interpreted as 0-step paths (clearly, the only player that can be reached from player i by following 0 edges is player i himself). To simplify the notation, I assume that $\boldsymbol{A}^0$ is the identity matrix even when $\boldsymbol{A} = \boldsymbol{0}$.

Therefore, $\boldsymbol{A}^k\mathbf{1}$ is a vector whose elements are the numbers of k-step paths from player i, which can be directly computed as $\boldsymbol{e}_i'\boldsymbol{A}^k\mathbf{1}$, where $\boldsymbol{e}_i$ is a column vector with the ith element equal to 1 and all other elements equal to 0. Similarly, $\mathbf{1}'\boldsymbol{A}^k\mathbf{1}$ is the number of all k-step paths in the network. The following expression performs these calculations for the network in example 3 (figure 2b), which has six players, six edges, and one two-edge path ($R \to D \to F$).

$$
\begin{array}{c}
\\
L \\ T \\ F \\ C \\ D \\ R \\ \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}
\end{array}
\begin{array}{c}
\begin{array}{cccc} \boldsymbol{A}^0\mathbf{1} & \boldsymbol{A}^1\mathbf{1} & \boldsymbol{A}^2\mathbf{1} & \boldsymbol{A}^3\mathbf{1} \end{array} \\
\begin{bmatrix}
1 & 3 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 2 & 1 & 0 \\
6 & 6 & 1 & 0
\end{bmatrix}
\end{array}.
$$

With this notation, I can now state the main result of this paper—the characterization theorem—which asserts that a unique equilibrium exists and shows how it is characterized using the components we have discussed.

Theorem 1. *There is a unique equilibrium, the final good price P^* is the solution to*

$$P^* - C = \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*), \tag{6}$$

and the individual prices are $p_i^* = c_i + \sum_{k=1}^{m} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*)$ for all i.

The proof in appendix A builds on the ideas discussed above. A few remarks are in order. Equation (6) combines all first-order conditions into one necessary condition for equilibrium. The uniqueness of its solution is straightforward to establish. Assumption 3 implies that each $g_k(P)$ is non-negative and weakly decreasing (this is formally shown in lemma 2 in appendix A). The right-hand side of equation (6) is therefore decreasing, whereas the left-hand side is strictly increasing.[16] The connection to inverted best-response functions is also clear, as the individual prices are determined by $p_i^* = c_i + f_i(P^*)$.

These arguments show that a unique combination of prices satisfies the first-order conditions of all firms, which is the only candidate for an equilibrium. To verify that this is indeed an equilibrium, note that we have identified a unique local optimum for each firm, and it gives a strictly positive profit for each firm. As the firms' profit functions are continuously differentiable and the profit at the corner solutions is non-positive, this must be a global maximizer. Thus, it is indeed an equilibrium outcome.

5 Multiple-marginalization Problem

Let me first interpret the equilibrium condition equation (6) by comparing it with the known benchmark cases. First, suppose that there is a single monopolist, i.e., $m = 1$ and

[16] If assumption 3 fails, equation (6) may not have a solution or may have multiple solutions. Moreover, without assumption 3, equation (6) is only necessary but not sufficient for equilibrium.

$\boldsymbol{A} = [0]$. Therefore, there is a single element on the right-hand side of (6) with value $g_1(P^*)$. We can rewrite the condition as

$$\frac{P^* - C}{P^*} = \frac{g_1(P^*)}{P^*} = \frac{1}{\varepsilon(P^*)}, \tag{7}$$

which is the standard inverse-elasticity rule: a markup (or Lerner (1934) index) equals the inverse elasticity. There is a usual monopoly distortion—as the monopolist does not internalize the impact on consumer surplus, the equilibrium price of the final good P is higher than the marginal cost C, and the equilibrium quantity is lower than the social optimum. It is also the joint profit-maximization outcome.

Second, consider $m > 1$ monopolists who are making their decisions simultaneously. That is, the network has m nodes but no edges. Analogously to the case with a single monopolist, we can then rewrite the equilibrium condition as

$$\frac{P^* - C}{P^*} = \mathbf{1}'\boldsymbol{A}^0\mathbf{1}\frac{g_1(P^*)}{P^*} = \frac{m}{\varepsilon(P^*)} > \frac{1}{\varepsilon(P^*)}. \tag{8}$$

The total markup is now strictly higher than in the case of a single monopolist. This is the *multiple-marginalization problem*—firms do not internalize the impact on consumer surplus, nor do they consider the impact on other firms. Therefore, the distortion is even larger than in the case of a single monopolist, which means that both total profits and social welfare are reduced compared to a single monopolist.[17]

Finally, the novel case studied in this paper involves multiple monopolists with some influences. That is, $m > 0$ and $\boldsymbol{A} \neq \mathbf{0}$. In this case, the condition can be written as

$$\frac{P^* - C}{P^*} = \frac{m}{\varepsilon(P^*)} + \sum_{k=2}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}\frac{g_k(P^*)}{P^*}. \tag{9}$$

The total markup, and therefore the distortion, is even higher than with m independent monopolists. The intuition for this is simple: suppose there is a single edge, so that firm i influences firm j. Then, in addition to the trade-offs firm i had before, raising the price now will reduce the profitability of firm j, who will respond by reducing its price. Therefore, p_i will be higher and p_j lower than with simultaneous decisions. How about the price of the final good, which depends on the sum of p_i and p_j? If the reduction in p_j were so large that the total price did not increase, then p_i would not be optimal, as the profit of firm i is $(p_i - c_i)D(P)$, i.e., increasing in p_i and decreasing in P, so firm i would want to raise the price even further. Thus, in equilibrium, the price of the final good should increase. I formalize and generalize this observation in corollary 1. The corollary follows from equation (6) and the non-negativity of g_k functions.

Corollary 1 (Magnified Multiple-marginalization Problem)**.** *Suppose that there are two networks $\boldsymbol{A}$ and $\boldsymbol{B}$ (both satisfying assumption 1) such that*

[17] There are two ways to think about multiple-marginalization. Since the seminal work by Spengler (1950), it has been mostly presented as a problem of sequential pricing. However, Cournot already observed almost 200 years earlier that two monopolists pricing perfect complements independently would distort the allocation more than a single monopolist pricing both (Sonnenschein, 1968). In this paper, I analyze both. I refer to the Cournot interpretation as multiple-marginalization and the Spengler version as magnified multiple-marginalization.

1. $\mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1} \geq \mathbf{1}'\boldsymbol{B}^{k-1}\mathbf{1}$ *for all* $k \in \{1, \ldots, m\}$ *and*

2. $\mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1} > \mathbf{1}'\boldsymbol{B}^{k-1}\mathbf{1}$ *for at least one* k*,*

then both social welfare and total profit in the case of $\boldsymbol{A}$ *is lower than with* $\boldsymbol{B}$*.*

For example, if $\boldsymbol{A}$ is obtained by adding influences to $\boldsymbol{B}$ (in a way that still satisfies assumption 1), then the assumption in corollary 1 is satisfied, and thus social welfare and joint profits are decreased.

The result shows that the multiple-marginalization problem is increased with strategic influences but does not quantify the magnitude of the increase. To illustrate that the impact may be severe, let me provide some numerical examples. First, suppose that demand is linear, $D(P) = 1-P$, there are no costs, and there are no price-takers. Standard calculations imply that the maximized total welfare would be $\frac{1}{2}$, and a single monopolist would choose a price of $\frac{1}{2}$, which would lead to a dead-weight loss of $\frac{1}{8}$. Therefore, with any network, the dead-weight loss is at least $\frac{1}{8}$ and at most $\frac{1}{2}$. Figure 4 illustrates the difference between the dead-weight loss in the best case (simultaneous decisions) and the worst case (sequential decisions). Even in the best case (blue line with triangles), the multiple-marginalization problem can be severe and increases with m. However, the distortions with strategic interactions (red line with circles) are much higher for any m, and the dead-weight loss approaches full destruction of social welfare quickly. This comparison shows that strategic influences magnify the multiple-marginalization problem for any m.

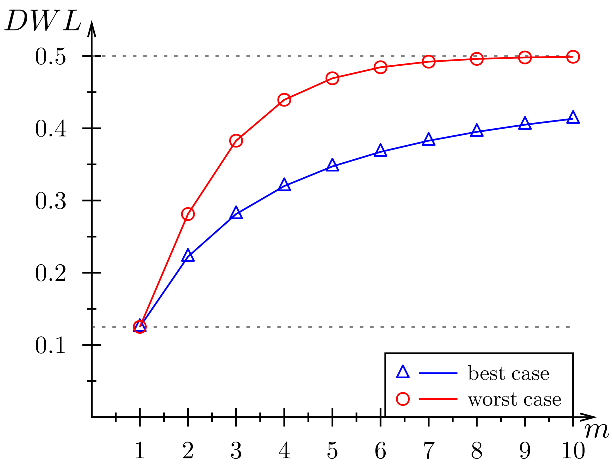


Figure 4: Example: comparison of dead-weight loss in the model with linear demand between the best case (simultaneous decisions) and the worst case (sequential decisions)

How much the number of firms matters compared to strategic influences depends on the shape of the demand function. If we perform the same calculation for a more general power demand function $D(P) = \sqrt[\beta]{a - bP}$, then when β is small, even with a large number of firms, the dead-weight losses from the best and worst case networks are approximately the same. However, when β is large, the difference is even larger than in the linear case.[18]

[18]In figure 4, with $m = 10$, the dead-weight loss in the best case is about 82.8% of the dead-weight loss in the worst case. When $\beta = \frac{1}{10}$, the same fraction is 99.5%, whereas when $\beta = 10$, the fraction is 34.1%.

This is natural, as β captures the decay of the importance of higher-order influences. When β is small, the direct influences are much more important, whereas when β is large, the higher-order influences matter more.

6 Influentiality

6.1 A Measure of Influentiality

All monopolists on the network have some market power and earn strictly positive profits. However, some firms are more influential than others. Which firms and how does this depend on the network? The answer comes directly from the characterization in theorem 1. For brevity, let me denote

$$I_i(\boldsymbol{A}) = \sum_{k=1}^{m} \boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1} g_k(P^*), \tag{10}$$

which is a sum of scalars $\boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1}$ weighted by $g_k(P^*)$. Note that $\boldsymbol{e}_i' \boldsymbol{A}^0 \mathbf{1} = 1$, $\boldsymbol{e}_i' \boldsymbol{A}^1 \mathbf{1}$ is the number of players i influences, $\boldsymbol{e}_i' \boldsymbol{A}^2 \mathbf{1}$ is the number of two-edge paths starting from i, and so on. Therefore, $I_i(\boldsymbol{A})$ can be interpreted as a measure of the influentiality of player i.

Fixing the equilibrium price of the final good P^*, the individual markups are $p_i^* - c_i = I_i(\boldsymbol{A})$, and therefore profits $\pi_i(\boldsymbol{p}^*) = (p_i^* - c_i) D(P^*) = I_i(\boldsymbol{A}) D(P^*)$. Thus, $I_i(\boldsymbol{A})$ fully captures the network details that affect firm i's action and payoff. Corollary 2 provides a formal statement.

Corollary 2 ($I_i(\boldsymbol{A})$ Summarizes Influences)**.** *$I_i(\boldsymbol{A}) > I_j(\boldsymbol{A})$ if and only if $\pi_i(\boldsymbol{p}^*) > \pi_j(\boldsymbol{p}^*)$ and $p_i - c_i > p_j^* - c_j$.*

This measure of influentiality $I_i(\boldsymbol{A})$ depends both on the network structure and the demand function. There are some cases when we can say more. In particular, if firm i has more influences in all levels than firm j, i.e. $\boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1} \geq \boldsymbol{e}_j' \boldsymbol{A}^{k-1} \mathbf{1}$ for all k and the inequality is strict for at least one k, then $I_i(\boldsymbol{A}) \geq I_j(\boldsymbol{A})$ regardless of the weights $g_k(P^*)$. The inequality is strict whenever $g_k(P^*) > 0$ for k such that $\boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1} > \boldsymbol{e}_j' \boldsymbol{A}^{k-1} \mathbf{1}$. For example, when firm i influences firm j, then $\boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1} \geq \boldsymbol{e}_j' \boldsymbol{A}^{k-1} \mathbf{1}$, and the inequality is strict for at least $k = 1$, so $I_i(\boldsymbol{A}) > I_j(\boldsymbol{A})$ with any demand function. The following corollary formalizes this observation.[19]

Corollary 3 (Early-Mover Advantage)**.** *If a_{ij} then $I_i(P^*) > I_j(P^*)$ and thus $\pi_i(\boldsymbol{p}^*) > \pi_j(\boldsymbol{p}^*)$ and $p_i^* - c_i > p_j^* - c_j$.*

Looking at the examples in previous sections, these corollaries have some immediate implications. Corollary 3 fully characterizes the influentiality in examples 1b–1c and 2, where for each pair of firms (i, j), either $a_{ij} = 1$ or $a_{ji} = 1$. In the example 1a, the two firms make simultaneous choices and obviously have the same influentiality and thus they choose the same dollar markups $p_i - c_i$ and get the same profit. This follows immediately

[19] This result generalizes the early-mover advantage result from Hinnosaar (2024).

from corollary 2. In example 4 (figure 3), where the choices are both simultaneous and sequential, corollary 2 still fully ranks the firms according to their influentiality, as firm 1 has two direct influences, firm 2 only one, and firms 3 and 4 have none. Therefore,

$$I_1(\boldsymbol{A}) > I_2(\boldsymbol{A}) > I_3(\boldsymbol{A}) = I_4(\boldsymbol{A}).$$

The remaining example 3 (figure 2b) illustrates that the influentiality measure is not only a function of the network but also the demand function. Note that $I_L(\boldsymbol{A}) = 3g_1(P^*)$, whereas $I_R(\boldsymbol{A}) = 2g_1(P^*)+g_2(P^*)$, so $I_L(\boldsymbol{A}) > I_R(\boldsymbol{A})$ if and only if $g_1(P^*) > g_2(P^*)$. This has an intuitive interpretation—if and only if indirect influences (in this case, $R \to D \to F$) are less important than direct influences (e.g., $L \to F$). In many network models, this is assumed to be the case for mathematical properties because when cycles are possible, otherwise the equilibrium is not guaranteed to exist. However, it is easy to find natural demand functions where this property is not satisfied. For example, with power demand $D(P) = d\sqrt[\beta]{a - bP}$, $g_k(P^*) = \beta^k(\overline{P} - P^*)$ where $\overline{P} = \frac{a}{b}$, so $g_1(P^*) > g_2(P^*)$ for any $\beta < 1$, and the opposite strict inequality holds for $\beta > 1$.

I will discuss the connection between the influentiality measure and the classic network centrality measures in the next section 6.2.

6.2 Connections with Network Centrality Measures

The measure of influentiality defined above is reminiscent of the classic measures of centrality, as they capture the same effects: a player is more influential if it influences either more players or more influential players. The difference is that while the classic centrality measures are defined purely using network characteristics, the influentiality measure defined here has endogenous weights that are determined by the model parameters such as the demand function, costs, and also by the price of the final good.

In some special cases, the connection is even closer. Consider again the case of power demand, $D(P) = d\sqrt[\beta]{a - bP}$. As I will show in section 7.2, this implies linear $g_k(P) = \beta^k(\overline{P} - P)$. Therefore, $I_i(\boldsymbol{A}) = (\overline{P} - P^*)B_i(\boldsymbol{A};\beta)$, where $B_i(\boldsymbol{A};\beta) = \sum_{k=1}^{m} \beta^k \boldsymbol{e}_i' \boldsymbol{A}^{k-1}\mathbf{1}$ is the Bonacich centrality measure of player i.[20] The general measure $I_i(\boldsymbol{A})$ can be thought of as a generalization of Bonacich centrality, where the weights are endogenously determined by the demand function and the equilibrium, rather than having exponential decay β^k.

However, the influence measure does not always have to resemble Bonacich centrality. Let me provide two more examples to show this. First, suppose $D(P) = de^{\sqrt{2(a-bP)}/b}$. This is a specifically constructed demand function, which implies $g(P) = g_1(P) = \sqrt{2(a - bP)}$ and therefore $g_2(P) = b$, which means that $g_k(P) = 0$ for all $k > 2$. With these weights, the influentiality measure simplifies to $I_i(\boldsymbol{A}) = \sqrt{2(a - bP^*)} + b\boldsymbol{e}_i'\boldsymbol{A}\mathbf{1}$, i.e., it depends only on the number of players directly influenced by player i. That is, the influentiality measure is a linear function of the degree centrality in this case.

[20] The textbook definition of the Bonacich centrality measure (Jackson, 2008) uses the expression $B_i(\boldsymbol{A};\beta) = [\mathbf{I} - \beta\boldsymbol{A}]^{-1}\mathbf{1}$. Given the acyclicity of the network in this paper, this definition coincides with the definition used above. Importantly, the standard Bonacich centrality measure requires $\beta < 1$ to ensure convergence. However, in acyclic networks, $\beta \geq 1$ can be allowed. This situation occurs naturally with many demand functions, such as the power demand with $\beta \geq 1$ mentioned in the previous section.

For another example, consider logit demand $D(P) = d\frac{e^{-\alpha P}}{1+e^{-\alpha P}}$. As I will show in section 7.3, it may lead to complex expressions, but when m is large enough, $g_1(P^*) \approx \frac{1}{\alpha}$ and $g_k(P^*) \approx 0$ for $k > 1$. Therefore, $I_i(\boldsymbol{A}) \approx \frac{1}{\alpha}$. This means that in the case of logit demand with sufficiently many players, the network structure does not affect the pricing of the individual firms. The relevant centrality measure is approximately a constant.

7 Computing the Equilibrium

In this subsection, I show how the equilibrium characterization can be used to compute the equilibrium and study some standard demand functions where the characterization is even simpler.

7.1 Linear Demand

Suppose that the demand function is linear $D(P) = a - bP$. Then $g(P) = -\frac{D(P)}{D'(P)} = \overline{P} - P = g_1(P)$ with $\overline{P} = \frac{a}{b}$, and therefore for all $k > 1$, $g_{k+1}(P) = -g_k'(P)g(P) = \overline{P} - P$. Equation (6) simplifies to

$$P^* - C = \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) = (\overline{P} - P^*)B(\boldsymbol{A}; 1), \tag{11}$$

where $B(\boldsymbol{A}; 1) = \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}$ is the sum of the number of influences at all levels, i.e., the number of players ($\mathbf{1}'\boldsymbol{A}^0\mathbf{1} = m$) plus the number of edges, plus the number of two-edge paths, and so on. Equation (11) is a linear equation, and its solution is the equilibrium price

$$P^* = \frac{C + \overline{P}B(\boldsymbol{A}; 1)}{1 + B(\boldsymbol{A}; 1)}. \tag{12}$$

As we would expect, increasing costs and increasing demand ($\overline{P} = \frac{a}{b}$ in particular) will raise the equilibrium price, but the pass-through is imperfect. Increasing the number of firms or connections between firms increases the equilibrium price through the marginalization effects discussed above. Similarly, we can compute the markups for individual firms,

$$p_i^* = c_i + \sum_{k=1}^{m} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) = c_i + \frac{B_i(\boldsymbol{A}; 1)}{1 + B(\boldsymbol{A}; 1)}(\overline{P} - C), \tag{13}$$

where $B_i(\boldsymbol{A}; 1) = \sum_{k=1}^{m} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}$ is the sum of influences of firm i, i.e., $\boldsymbol{e}_i'\boldsymbol{A}^0\mathbf{1} = 1$ ("influencing" oneself) plus $\boldsymbol{e}_i'\boldsymbol{A}^1\mathbf{1} =$ number of players i influences plus the number of paths starting from i. By construction $B(\boldsymbol{A}; 1) = \sum_{i=1}^{m} B_i(\boldsymbol{A}; 1)$.

Consider again example 3 (figure 2b), for which the corresponding $\boldsymbol{A}^{k-1}\mathbf{1}$ terms are computed in section 4.3. Suppose that $D(P) = 1 - P$, with no costs. Then $B(\boldsymbol{A}; 1) = 6 + 6 + 1 = 13$, and therefore $P^* = \frac{B(\boldsymbol{A};1)}{1+B(\boldsymbol{A};1)} = \frac{13}{14}$. Similarly, individual prices are $p_i^* = \frac{B_i(\boldsymbol{A};1)}{1+B(\boldsymbol{A};1)}$. For example, $p_L^* = \frac{4}{14}$, $p_T^* = p_F^* = p_C^* = \frac{1}{14}$, $p_D^* = \frac{2}{14}$, and $p_R^* = \frac{4}{14}$. In particular, observe that $p_L^* = p_R^*$, but for different reasons—firm L influences three firms directly, whereas R influences two firms directly and one indirectly. In the case of linear demand, these two types of influences are weighted equally.

7.2 Power Demand

The calculations are similar for a more general power demand $D(P) = d\sqrt[\beta]{a - bP}$. Then $g(P) = \beta(\overline{P} - P)$ with $\overline{P} = \frac{a}{b}$, and therefore $g_k(P) = \beta^k(\overline{P} - P)$, so that equation (6) gives the same expression for the equilibrium price of the final good

$$P^* = \frac{C + \overline{P}B(\boldsymbol{A};\beta)}{1 + B(\boldsymbol{A};\beta)} \tag{14}$$

but now $B(\boldsymbol{A};\beta) = \sum_{k=1}^{m} \beta^k \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}$, i.e., the influences at various levels are weighted by $1, \beta, \beta^2, \ldots$. Here, β can be interpreted as a decay or discount factor for more indirect influences.[21] Similarly, for individual firms,

$$p_i^* = c_i + \frac{B_i(\boldsymbol{A};\beta)}{1 + B(\boldsymbol{A};\beta)}(\overline{P} - C), \tag{15}$$

where $B_i(\boldsymbol{A};\beta) = \sum_{k=1}^{m} \beta^k \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}$, i.e., influences are again weighted by factor β^k.

7.3 Logit Demand

Take logit demand $D(P) = d\frac{e^{-\alpha P}}{1+e^{-\alpha P}}$ with $\alpha > 0$. Then $g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right]$. Let us first consider the example discussed in previous subsections to illustrate how the characterization result could be used for more complicated demand functions. Suppose again that $C = 0$ and the network from example 3 (figure 2b), with $C = 0$. As the depth of the network is $d(\boldsymbol{A}) = 3$, we need to compute functions

$$\begin{aligned}
g_1(P) &= g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right], \\
g_2(P) &= -g_1'(P)g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right]e^{-\alpha P}, \\
g_3(P) &= -g_2'(P)g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right]e^{-\alpha P}\left[1 + 2e^{-\alpha P}\right].
\end{aligned}$$

The equilibrium condition (6) takes the form $P^* = 6g_1(P^*) + 6g_2(P^*) + g_3(P^*)$, which is straightforward to solve numerically. For example, when $\alpha = 1$, we get

$$P^* = 6 + 13e^{-P^*} + 9e^{-2P^*} + 2e^{-3P^*},$$

implying $P^* \approx 6.0313$ and individual prices $p_L^* \approx 1.0096, p_T^* = p_F^* = p_C^* \approx 1.0024, p_D^* \approx 1.0048$, and $p_R^* \approx 1.0096$.

The numerical results point to a specific equilibrium property with logit demand—all prices are only slightly above 1. Inspecting the $g_k(P)$ functions above reveals the reason. Specifically, the term $e^{-\alpha P^*}$ converges to zero as P^* increases. Therefore, for sufficiently large P^*, the weight $g_1(P^*)$ converges to a constant $\frac{1}{\alpha}$, whereas the weights $g_k(P^*)$ for $k > 1$ converge to zero. Thus, if the equilibrium price P^* is large, it is almost solely determined by the number of players. Lemma 1 formalizes this observation.

[21] Note that $\beta > 0$ (as otherwise demand would not be decreasing), but it can be greater or less than 1. In fact, when $\beta = 1$, the demand function is linear, so that $B(\boldsymbol{A};\beta) = B(\boldsymbol{A};1)$.

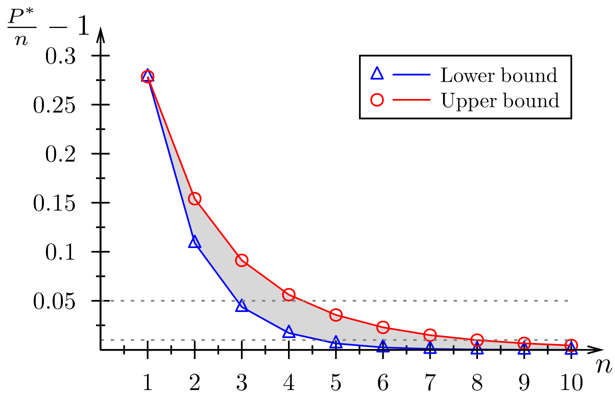


Figure 5: Bounds for equilibrium prices with logit demand $D(P) = \frac{e^{-P}}{1+e^{-P}}$ and $C = 0$ depending on the number of firms.

Lemma 1 (Approximate Equilibrium with Logit Demand)**.** *With logit demand $D(P) = d\frac{e^{-\alpha}}{1+e^{-\alpha P}}$, the price of final good P^* and individual prices p_i^* satisfy the following conditions*

1. $P^* > C + \frac{m}{\alpha}$ *and* $p_i^* > c_i + \frac{1}{\alpha}$ *for all* i*,*
2. $P^* = C + \frac{m}{\alpha} + O\left(\left[\frac{2}{e}\right]^m\right)$ *and* $p_i^* = c_i + \frac{1}{\alpha} + O\left(\left[\frac{2}{e}\right]^m\right)$ *for all* i*.*[22]

Lemma 1 implies that when m is large enough, $P^* \approx C + \frac{m}{\alpha}$ and each $p_i^* \approx c_i + \frac{1}{\alpha}$. This is a limit result, but as we saw from the example above, the approximation with $m = 6$ seems already quite precise. Figure 5 illustrates that the convergence is indeed fast. It shows that while for small numbers of players, there is a difference between the lower bound (simultaneous decisions) and the upper bound (sequential decisions), the difference shrinks quickly and becomes negligible with 5–10 players. In particular, the figure illustrates that $\frac{1}{m}\left[P^* - C - \frac{m}{\alpha}\right] \approx 0$ for any network with about ten players or more.

8 Discussion

This paper characterizes the equilibrium behavior for a general class of price-setting games on a network. Under regularity assumptions, there is a unique equilibrium, which is straightforward to compute even with non-linear demand functions and complex networks. For the most common demand functions, such as linear, power, and logit demand, I provide even simpler characterization results.

The key distortion is multiple-marginalization. A novel finding of the paper is the magnification of the marginalization problem: strategic interactions on the network lead to an even bigger marginalization problem. Firms set too high markups not only because they do not internalize the negative impact on consumer surplus and other firms' profits but also because they benefit from discouraging other firms from setting high markups.

The results define a natural measure of influentiality that ranks firms according to their markups and profits. Firms are more influential if they influence more firms or more

[22]Where $f(m) = O(g(m))$ means that $\limsup_{m\to\infty} \left|\frac{f(x)}{g(x)}\right| < \infty$.

influential firms. In some special cases, the influentiality measure simplifies to standard measures of centrality. I provide examples where it takes the form of Bonacich centrality, degree centrality, or is independent of the network structure.

The results of this paper are quite general in terms of network structure and demand functions, but I make several simplifying assumptions in other areas. For example, I assume constant marginal costs, which provides the model with tractability. Additionally, I model the competition in an extreme way, with firms either being monopolists. The model can be easily extended to situations where some inputs are produced by price-takers. It would be interesting to study intermediate forms of imperfect competition.

In this paper, the analysis is described in terms of price setting on a network that supplies a single final product. There are other applications fitting the same mathematical model. An obvious example is multiple monopolists selling perfect complements. More generally, the model applies whenever multiple players choose actions so that their payoffs depend linearly on their own actions, the marginal benefit is a decreasing function of the total action, and the actions are (higher-order) strategic substitutes. For example, private provision of public goods and contests satisfy this general description.

Finally, the results have significant policy implications, which I have not discussed so far. Policy decisions impact the network structure of production and thus influence the network of influences discussed in this paper. For example, in merger analysis, vertical mergers are typically considered socially desirable, as they reduce the number of firms and, thus, the marginalization problem. However, in situations where a merger leads to a new network of influences where the merged firm has stronger commitment power and thus more indirect influences, a vertical merger can also reduce social welfare due to magnified multiple-marginalization.[23]

References

Acemoglu, D. and M. K. Jensen (2013): "Aggregate Comparative Statics," *Games and Economic Behavior*, 81, 27–49.

Amarasinghe, A., P. A. Raschky, Y. Zenou, and J. Zhou (2023): "Conflicts in Networks," *Manuscript.*

Anderson, S. P., N. Erkal, and D. Piccinin (2020): "Aggregative Games and Oligopoly Theory: Short-Run and Long-Run Analysis," *RAND Journal of Economics*, 51, 470–495.

Antràs, P. and A. Costinot (2011): "Intermediated Trade," *Quarterly Journal of Economics*, 126, 1319–1374.

Atalay, E., A. Hortaçsu, J. Roberts, and C. Syverson (2011): "Network Structure of Production," *Proceedings of the National Academy of Sciences*, 108, 5199–5202.

Ballester, C., A. Calvó-Armengol, and Y. Zenou (2006): "Who's Who in Networks. Wanted: The Key Player," *Econometrica*, 74, 1403–1417.

[23] A formal discussion of this scenario is available in the working paper version of the paper.

Bimpikis, K., S. Ehsani, and R. İlkiliç (2019): "Cournot Competition in Networked Markets," *Management Science*.

Bloch, F. and N. Quérou (2013): "Pricing in Social Networks," *Games and Economic Behavior*, 80, 243–261.

Bolton, P. and M. D. Whinston (1993): "Incomplete Contracts, Vertical Integration, and Supply Assurance," *Review of Economic Studies*, 60, 121–148.

Bramoullé, Y. and R. Kranton (2007): "Public Goods in Networks," *Journal of Economic Theory*, 135, 478–494.

Bramoullé, Y., R. Kranton, and M. D'Amours (2014): "Strategic Interaction and Networks," *American Economic Review*, 104, 898–930.

Buehler, S. and D. L. Gärtner (2013): "Making Sense of Nonbinding Retail-Price Recommendations," *American Economic Review*, 103, 335–359.

Calvó-Armengol, A., E. Patacchini, and Y. Zenou (2009): "Peer Effects and Social Networks in Education," *Review of Economic Studies*, 76, 1239–1267.

Chaney, T. (2014): "The Network Structure of International Trade," *American Economic Review*, 104, 3600–3634.

Choi, S., A. Galeotti, and S. Goyal (2017): "Trading in Networks: Theory and Experiments," *Journal of the European Economic Association*, 15, 784–817.

Choi, S. C. (1991): "Price Competition in a Channel Structure with a Common Retailer," *Marketing Science*, 10, 271–296.

Condorelli, D., A. Galeotti, and L. Renou (2017): "Bilateral Trading in Networks," *Review of Economic Studies*, 84, 82–105.

Cortes-Corrales, S. and P. M. Gorny (2024): "How Strength Asymmetries Shape Multi-Sided Conflicts," *Economic Theory*.

Costinot, A., J. Vogel, and S. Wang (2013): "An Elementary Theory of Global Supply Chains," *Review of Economic Studies*, 80, 109–144.

Crawford, G. S., R. S. Lee, M. D. Whinston, and A. Yurukoglu (2018): "The Welfare Effects of Vertical Integration in Multichannel Television Markets," *Econometrica*, 86, 891–954.

Dziubiński, M., S. Goyal, and D. E. N. Minarsch (2021): "The strategy of conquest," *Journal of Economic Theory*, 191, 105161.

Euler, L. (1783): "De serie Lambertina Plurimisque eius insignibus proprietatibus," *Acta Academiae scientiarum imperialis petropolitanae*, 29–51.

Fainmesser, I. P. and A. Galeotti (2016): "Pricing Network Effects," *Review of Economic Studies*, 83, 165–198.

Farrell, J. and C. Shapiro (1990): "Horizontal Mergers: An Equilibrium Analysis," *American Economic Review*, 80, 107–126.

Franke, J. and T. Öztürk (2015): "Conflict Networks," *Journal of Public Economics*, 126, 104–113.

Galeotti, A., B. Golub, S. Goyal, E. Talamàs, and O. Tamuz (2021): "Taxes and Market Power: A Network Approach," .

Galeotti, A., S. Goyal, M. O. Jackson, F. Vega-Redondo, and L. Yariv (2010): "Network Games," *Review of Economic Studies*, 77, 218–244.

Gayle, P. G. (2013): "On the Efficiency of Codeshare Contracts between Airlines: Is Double Marginalization Eliminated?" *American Economic Journal: Microeconomics*, 5, 244–273.

Goyal, S., H. Heidari, and M. Kearns (2019): "Competitive Contagion in Networks," *Games and Economic Behavior*, 113, 58–79.

Grossman, S. J. and O. D. Hart (1986): "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 94, 691–719.

Hinnosaar, T. (2024): "Optimal Sequential Contests," *Theoretical Economics*, 19, 207–244.

Jackson, M. O. (2008): *Social and Economic Networks*, Princeton University Press, published: Hardcover.

Jackson, M. O. and Y. Zenou (2015): "Games on Networks," in *Handbook of Game Theory With Economic Applications*, Elsevier, vol. 4, 95–163.

Jensen, M. K. (2010): "Aggregative Games and Best-Reply Potentials," *Economic Theory*, 43, 45–66.

Jeuland, A. P. and S. M. Shugan (1988): "Note—Channel of Distribution Profits When Channel Members Form Conjectures," *Marketing Science*, 7, 202–210, publisher: INFORMS.

Kahana, N. and D. Klunover (2018): "Sequential Lottery Contests With Multiple Participants," *Economics Letters*, 163, 126–129.

Kovenock, D. and B. Roberson (2018): "The Optimal Defense of Networks of Targets," *Economic Inquiry*, 56, 2195–2211.

Kuhn, K.-U. and X. Vives (1999): "Excess Entry, Vertical Integration, and Welfare," *RAND Journal of Economics*, 30, 575–603.

Lerner, A. P. (1934): "The Concept of Monopoly and the Measurement of Monopoly Power," *The Review of Economic Studies*, 1, 157–175.

Liu, E. (2019): "Industrial Policies in Production Networks," *Quarterly Journal of Economics*, 134, 1883–1948.

Luco, F. and G. Marshall (2020): "The Competitive Impact of Vertical Integration by Multiproduct Firms," *American Economic Review*, 110, 2041–2064.

Manea, M. (2011): "Bargaining in Stationary Networks," *American Economic Review*, 101, 2042–2080.

——— (2018): "Intermediation and Resale in Networks," *Journal of Political Economy*, 126, 1250–1301.

Martimort, D. and L. Stole (2012): "Representing Equilibrium Aggregates in Aggregate Games With Applications to Common Agency," *Games and Economic Behavior*, 76, 753–772.

Mathewson, G. F. and R. Winter (1984): "An Economic Theory of Vertical Restraints," *RAND Journal of Economics*, 15, 27–38.

Matros, A. and D. Rietzke (2018): "Contests on Networks," *manuscript*.

Matutes, C. and P. Regibeau (1992): "Compatibility and Bundling of Complementary Goods in a Duopoly," *Journal of Industrial Economics*, 40, 37–54.

Nava, F. (2015): "Efficiency in Decentralized Oligopolistic Markets," *Journal of Economic Theory*, 157, 315–348.

Noble, P. M. and T. S. Gruca (1999): "Industrial Pricing: Theory and Managerial Practice," *Marketing Science*, 18, 435–454.

Nocke, V. and N. Schutz (2018): "Multiproduct-Firm Oligopoly: An Aggregative Games Approach," *Econometrica*, 86, 523–557.

Nocke, V. and L. White (2007): "Do Vertical Mergers Facilitate Upstream Collusion?" *American Economic Review*, 97, 1321–1339.

Oberfield, E. (2018): "A Theory of Input-Output Architecture," *Econometrica*, 86, 559–589.

Ordover, J. A., G. Saloner, and S. C. Salop (1990): "Equilibrium Vertical Foreclosure," *American Economic Review*, 80, 127–142.

Pellegrino, B. (2023): "Product Differentiation and Oligopoly: A Network Approach," *Manuscript*.

Rey, P. and J. Tirole (1986): "The Logic of Vertical Restraints," *American Economic Review*, 76, 921–939.

Riordan, M. H. (1998): "Anticompetitive Vertical Integration by a Dominant Firm," *American Economic Review*, 88, 1232–1248.

Salinger, M. A. (1988): “Vertical Mergers and Market Foreclosure,” *Quarterly Journal of Economics*, 103, 345–356.

——— (1989): “The Meaning of "Upstream" and "Downstream" and the Implications for Modeling Vertical Mergers,” *Journal of Industrial Economics*, 37, 373–387.

Selten, R. (1970): *Preispolitik der Mehrproduktenunternehmung in der statischen Theorie*, Springer-Verlag.

Sonnenschein, H. (1968): “The Dual of Duopoly Is Complementary Monopoly: or, Two of Cournot’s Theories Are One,” *Journal of Political Economy*, 76, 316–318.

Spencer, B. J. and R. W. Jones (1991): “Vertical Foreclosure and International Trade Policy,” *Review of Economic Studies*, 58, 153–170.

Spengler, J. J. (1950): “Vertical Integration and Antitrust Policy,” *Journal of Political Economy*, 58, 347–352.

Wang, J.-C., A. H.-L. Lau, and H.-S. Lau (2013): “Dollar vs. Percentage Markup Pricing Schemes Under a Dominant Retailer,” *European Journal of Operational Research*, 227, 471–482.

Zhou, J. and Y.-J. Chen (2015): “Key Leaders in Social Networks,” *Journal of Economic Theory*, 157, 212–235.

A Proofs

A.1 Proof of proposition 1

Proof. In each case, I directly verify the claim:

1. Linear demand is a special case of power demand with $d = \beta = 1$.
2. Power demand implies $g(P) = -\frac{d(a-bP)^{\frac{1}{\beta}}}{d\frac{1}{\beta}(a-bP)^{\frac{1}{\beta}-1}(-b)} = \beta(\overline{P} - P)$, where $\overline{P} = \frac{a}{b}$. Then $-g'(P) = \beta > 0$ and $(-1)^k \frac{d^k g(P)}{dP^k} = 0$, for all $k > 1$.
3. Logit demand implies $g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right]$. Then $(-1)^k \frac{d^k g(P)}{dP^k} = \alpha^{k-1} e^{-\alpha P} > 0$.
4. Exponential demand implies $g(P) = \frac{1}{\alpha}\left[\overline{P} e^{-\alpha P} - 1\right]$. Therefore $(-1)^k \frac{d^k g(P)}{dP^k} = \alpha^{k-1} \overline{P} e^{-\alpha P} > 0$.

□

A.2 Proof of theorem 1

Before the proof, let me introduce some useful notation. Each player $i \in \mathcal{N} = \{1, \ldots, m\}$, observes prices of some players. Let the set of these players be $\mathcal{O}_i = \{j : a_{ji} = 1\} \subset \mathcal{N}$ (possibly empty set) and vector of these prices $\boldsymbol{p}_i = (p_j)_{j \in \mathcal{O}_i}$. Player i's strategy is $p_i^*(\boldsymbol{p}_i)$. Player i also influences some players, let the set of these players be $\mathcal{I}_i = \{j : a_{ij} = 1\} \subset \mathcal{N}$ (again, possibly empty). Each such player $j \in \mathcal{I}_i$ uses the equilibrium strategy $p_j^*(\boldsymbol{p}_j)$. By definition, $i \in \mathcal{O}_j$, i.e., p_i is one of the inputs in $\boldsymbol{p}_j$. However, i does not necessarily observe all prices in $\boldsymbol{p}_j$, therefore it must make an equilibrium conjecture about these values. Let $p_j^i(p_i, \boldsymbol{p}_i)$ denote player j's action as seen by player i. That is, $p_j^i(p_i, \boldsymbol{p}_i) = p_j^*(\boldsymbol{p}_j')$, where $\boldsymbol{p}_j = (p_k')_{k \in \mathcal{O}_j}$ is such that $p_k' = p_k$ if $k \in \mathcal{O}_i$ or $k = i$ and $p_k' = p_k^i(p_i, \boldsymbol{p}_i)$ otherwise. The last step makes the definition recursive, but it is well-defined, as each such step strictly reduces the number of arguments in the function. Finally, there are also some players whose prices that i neither observes nor influences, let this set be $\mathcal{U}_j = \{j : a_{ji} = a_{ij} = 0\} \subset \mathcal{N}$. For these players, i expects the actions to be $p_j^i(\boldsymbol{p}_i)$ defined in the same way as above, but its arguments do not include p_i.

Using this notation, a firm i that observes $\boldsymbol{p}_i$ and sets its price to p_i, expects the price of the final good to be

$$P^i(p_i|\boldsymbol{p}_i) = c_0 + p_i + \sum_{j \in \mathcal{O}_i} p_j + \sum_{j \in \mathcal{I}_i} p_j^i(p_i, \boldsymbol{p}_i) + \sum_{j \in \mathcal{U}_i} p_j^i(\boldsymbol{p}_i). \tag{16}$$

The main idea in the proof is the following. Instead of choosing price p_i to maximize profit $(p_i - c_i)D(P^i(p_i|\boldsymbol{p}_i))$, we can think of player i choosing the final good price P to induce. For this, let me assume that in the relevant range, $P^i(p_i|\boldsymbol{p}_i)$ is smooth and strictly increasing in p_i, so that it has a differentiable and strictly increasing inverse function $f_i(P|\boldsymbol{p}_i)$ such that $P^i(f_i(P|\boldsymbol{p}_i)|\boldsymbol{p}_i) = P$. Then the maximization problem is

$$\max_P [f_i(P|\boldsymbol{p}_i) - c_i]D(P),$$

which leads to first-order condition $f_i'(P|\boldsymbol{p}_i)D(P) + [f_i(P|\boldsymbol{p}_i) - c_i]D'(P) = 0$ or equivalently

$$f_i(P|\boldsymbol{p}_i) - c_i = g(P)f_i'(P|\boldsymbol{p}_i). \tag{17}$$

Note that there is one-to-one mapping between representing equilibrium behavior in terms of functions $f_i(P|\boldsymbol{p}_i)$ and in terms of $p_i^*(\boldsymbol{p}_i)$.

Proof. Observe that the equilibrium must be interior, i.e., each $c_i < p_i < \overline{P}$ for each firm. If this is not the case for the firm i, then its equilibrium profit is non-positive. This could be for one of two reasons. First, the equilibrium price of the final good is so high that $D(P) = 0$. In this case, all equilibrium profits are non-positive and there must be at least one firm i who, by reducing its price (and anticipating the responses of firms influenced), can make the final good price low enough so that it ensures a strictly positive profit. This would be a profitable deviation. Second, if $P < \overline{P}$ and $p_i \leq c_i$, then firm i can raise its price slightly and increase its profit.

I will first derive necessary conditions for an interior equilibrium and combine them into one necessary condition, which gives equation (6). I then show that it has a unique solution and finally verify that it is indeed an equilibrium by verifying that each firm indeed chooses a price that maximizes its profit.

Let us start with any player i who does not influence any other players, i.e., $\boldsymbol{e}_i'\boldsymbol{A}\mathbf{1} = 0$ or equivalently $\mathcal{I}_i = \varnothing$. Then we can rewrite equation (16) as

$$P = c_0 + f_i(P|\boldsymbol{p}_i) + \sum_{j\in\mathcal{O}_i} p_j + \sum_{j\in\mathcal{U}_i} p_j^i(\boldsymbol{p}_i). \tag{18}$$

Differentiating this expression with respect to P shows that $f_i'(P|\boldsymbol{p}_i) = 1$ (that is, player i can raise the price of the final good by ε by raising its own price by ε). Therefore equation (17) implies $f_i(P|\boldsymbol{p}_i) = c_i + g(P)$. Note that this expression is independent of $\boldsymbol{p}_i$, so I can drop it as an argument for f_i and write simply as $f_i(P) = c_i + g(P)$.

Let us take now any player i and suppose that the optimal behavior of all players $j \in \mathcal{I}_i$ is described corresponding functions $f_j(P)$ that do not depend on the remaining arguments $\boldsymbol{p}_j$. Then we can rewrite equation (16) as

$$P = c_0 + f_i(P|\boldsymbol{p}_i) + \sum_{j\in\mathcal{O}_i} p_j + \sum_{j\in\mathcal{I}_i} f_j(P) + \sum_{j\in\mathcal{U}_i} p_j^i(\boldsymbol{p}_i). \tag{19}$$

Differentiating this expression and inserting it to equation (17) gives

$$f_i'(P|\boldsymbol{p}_i) = 1 - \sum_{j\in\mathcal{I}_i} f_j'(P) \quad\Rightarrow\quad f_i(P|\boldsymbol{p}_i) = g(P)\left[1 - \sum_{j\in\mathcal{I}_i} f_j'(P)\right]. \tag{20}$$

This expression is again independent of the arguments $\boldsymbol{p}_i$, which we can therefore drop. Moreover, these arguments give precise analytic expressions for $f_i(P)$ functions. We already saw that $f_i(P) = g(P) = \sum_{k=1}^m \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P)$ when $\boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1} = 0$ for all $k > 1$ (i.e., players who do not influence anybody). Suppose that every player $j \in \mathcal{I}_i$ has

$$f_j(P) - c_j = \sum_{k=1}^m \boldsymbol{e}_j'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P). \tag{21}$$

Then for player i we must have

$$f_i(P) - c_i = g(P)\left[1 - \sum_{j\in\mathcal{I}_i} f_j'(P)\right] = \underbrace{g(P)}_{=\boldsymbol{e}_i'\boldsymbol{A}^0\mathbf{1}g_1(P)} + \sum_{k=1}^m \underbrace{[-g_k'(P)g(P)]}_{g_{k+1}(P)}, \tag{22}$$

which, after change of variables from k to $k-1$ and combining the terms, gives the same expression as in equation (21).[24]

Therefore on-path, when the equilibrium price of the final good is P^*, the individual prices are indeed given by the expressions in the theorem. The price of the final good must be sum of all the input prices, therefore P^* must satisfy

$$P^* = c_0 + \sum_{i\in\mathcal{N}} f_i(P^*) = \underbrace{c_0 + \sum_{i\in\mathcal{N}} c_j}_{=C} + \sum_{k=1}^m \underbrace{\sum_{i\in\mathcal{N}} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}}_{=\mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}}\, g_k(P^*),$$

which gives the equation (6).

[24]Note that no player can have level-m influences, i.e., $\boldsymbol{e}_i'\boldsymbol{A}^m\mathbf{1} = 0$.

Below I prove two technical lemmas (lemmas 2 and 3) provide monotonicity properties that imply existence and uniqueness of equilibria. We can rewrite equation (6) as $f(P) = P - C - \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P) = 0$. At $P = 0$ we have $f(0) = -C - \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(0) < 0$ and $\lim_{P\to\overline{P}} f(P) > 0$. By lemma 3, function $f(P)$ is strictly increasing and therefore $f(P) = 0$ has a unique solution, which is the equilibrium price of the final good $P^* \in (0, \overline{P})$.

Next, in the argument above, we assumed that the inverse function of $f_i(P)$ function is strictly increasing. The construction implied a necessary condition that $f_i(P)$ must satisfy and lemma 3 shows that it implies that $f_i(P)$ is indeed strictly increasing, therefore the inverse function $P^i(p_i|\boldsymbol{p}_i)$ is indeed a well-defined strictly increasing function. Finally, to verify that the solution we found is indeed an equilibrium, we need to verify that the solution we derived is indeed a global maximizer for each firm. Notice that by lemma 3, the optimality condition equation (17) has a unique solution for each firm. Therefore we have identified a unique local optimum for each firm. As we already verified that corner solutions would give non-positive profits for each firm and the interior solution gives strictly positive profit, this must be a global maximizer.

Lemma 2 (Monotonicity of $g_k(P)$)**.** *$g_k(P)$ is $(d(\boldsymbol{A}) + 1 - k)$-times monotone.*

Proof. $g_1(P) = g(P) = -\frac{D(P)}{D'(P)}$ is $d(\boldsymbol{A})$-times monotone by assumption 3. Therefore $g'(P)$ is $(d(\boldsymbol{A}) - 1)$-times and $g_2(P) = -g_1'(P)g(P)$ is $(d(\boldsymbol{A}) - 1)$-times monotone. The rest follows by induction in the same way, if $g_k(P)$ is $(d(\boldsymbol{A}) + 1 - k)$-times monotone, then $g_{k+1}(P) = -g_k'(P)g(P)$ is $(d(\boldsymbol{A}) - k)$-times monotone. ☐

Lemma 3 (Monotonicity of $f(P), f_i(P)$)**.** *The following monotonicity properties hold*

1. *$f(P) = P - C - \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P)$ is strictly increasing,*

2. *$f_i(P) = c_i - \sum_{k=1}^{m} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P)$ is strictly increasing for each $i \in \{1, \dots, m\}$,*

3. *$f_i'(P)g(P) = \sum_{k=1}^{m} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P)$ is (weakly) decreasing for each $i \in \{1, \dots, m\}$.*

Proof. Each $\mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}$ and $\boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}$ is a non-negative integer and each $g_k(P)$ weakly decreasing in $-P$ by lemma 2, which implies weak monotonicity of $f_i'(P)g(P)$. Moreover, when $k = 1$, then $g_1(P) = g(P)$ which is strictly decreasing by assumption 3 and $\boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1} = 1 > 0$, which implies that $f_i(P)$ is strictly increasing. As $P - C$ is strictly increasing, then $f(P)$ is also strictly increasing. ☐

☐

A.3 Proof of lemma 1

Remark: The equilibrium prices in our model, denoted as P^* and p_i^*, are determined by all parameters of the model. In particular, the equilbrium prices depend on the number of monopolists (m), their costs (c_i), and the network's structure ($\boldsymbol{A}$).

Proof. Using the facts that $g(P) = \frac{1}{\alpha}\left[1 + e^{-\alpha P}\right] > \frac{1}{\alpha}$ and $g_k(P) > 0$ for all $k > 0$, equation (6) gives $P^* = C + \sum_{k=1}^{m} \mathbf{1}'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) > C + mg(P^*) > C + \frac{m}{\alpha}$. Similarly for individual prices, $p_i^* = c_i + \sum_{k=1}^{m} \boldsymbol{e}_i'\boldsymbol{A}^{k-1}\mathbf{1}g_k(P^*) > c_i + g(P^*) > c_i + \frac{1}{\alpha}$.

Using the lower bound for P^*, we can bound $e^{-\alpha P^*} < e^{-\alpha C - \alpha \frac{m}{\alpha}} = e^{-\alpha C} e^{-m}$. Therefore $e^{-\alpha P^*} = O(e^{-m})$. I use this result to prove lemma 4 that shows that $g_1(P^*) = \frac{1}{\alpha} + O(e^{-m})$ and $g_k(P^*) = O(e^{-m})$ for all $k > 1$. Using this, we can define $G^m(P^*)$ as follows

$$G^m(P^*) \equiv \max \left\{ g_1(P^*) - \frac{1}{\alpha}, g_2(P^*), \ldots, g_m(P^*) \right\} = O(e^{-m}).$$

Therefore equation (6) gives

$$P^* \leq C + \frac{m}{\alpha} + \sum_{k=1}^{m} \mathbf{1}' \boldsymbol{A}^{k-1} \mathbf{1} G^m(P^*) = C + \frac{m}{\alpha} + O(e^{-m}) B(\boldsymbol{A}; 1), \tag{23}$$

where $B(\boldsymbol{A}; 1) = \sum_{k=1}^{m} \mathbf{1}' \boldsymbol{A}^{k-1} \mathbf{1}$. Now, note that $B(\boldsymbol{A}; 1)$ increases each time an edge is added to $\boldsymbol{A}$, so its upper bound is when the network is most connected (fully sequential decisions) and lower bound with least connected network (simultaneous decisions), so that $m \leq B(\boldsymbol{A}; 1) \leq 2^m - 1$. Therefore $B(\boldsymbol{A}; 1) = O(2^m)$. Inserting this observation to previous expression gives $P^* = C + \frac{m}{\alpha} + O\left(\left[\frac{2}{e}\right]^m\right)$. Finally, for the equilibrium expression for individual prices is

$$p_i^* = c_i + \sum_{k=1}^{m} \boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1} g_k(P^*) = c_i + \frac{1}{\alpha} + O(e^{-m}) B_i(\boldsymbol{A}; 1), \tag{24}$$

where $B_i(\boldsymbol{A}) = \sum_{k=1}^{m} \boldsymbol{e}_i' \boldsymbol{A}^{k-1} \mathbf{1}$, which is by the same arguments as above $B_k(\boldsymbol{A}; 1) = O(2^m)$ and therefore $p_i = c_i + \frac{1}{\alpha} + O\left(\left[\frac{2}{e}\right]^m\right)$. $\square$

Lemma 4. *With logit demand $D(P) = d \frac{e^{-\alpha}}{1+e^{-\alpha P}}$, functions $g_k(P)$ and their derivatives have the following limit properties at $P = P^*$*

1. $g_1(P^*) = \frac{1}{\alpha} + O(e^{-m}) = O(1)$, $g_k(P^*) = O(e^{-m})$ *for all* $k \in \{2, \ldots, m\}$,
2. $\frac{d^l g_k(P^*)}{dP^l} = O(e^{-m})$ *for all* $k, l \in \{1, \ldots, m\}$.

Proof. We showed that $e^{-\alpha P^*} = O(e^{-m})$. Consider $g_1(P^*)$ first. We get $g_1(P^*) = g(P^*) = \frac{1}{\alpha} + \frac{1}{\alpha} e^{-\alpha P^*} = \frac{1}{\alpha} + O(e^{-m}) = O(1)$. Therefore, $\frac{d^l g_1(P^*)}{dP^l} = -(-\alpha)^{l-1} e^{-\alpha P^*} = O(e^{-m})$.

The rest of the proof is by induction. Suppose that the claim holds for $g_1, \ldots, g_k$. Now, $g_{k+1}(P^*) = -g_k'(P^*) g(P^*) = O(e^{-m})$ as $g(P^*) = O(1)$ and $g_k'(P^*) = O(e^{-m})$ by induction assumption. Each derivative can be written as

$$\frac{d^l g_{k+1}(P^*)}{dP^l} = -\sum_{j=0}^{l} \binom{l}{j} g_k^{(l-j+1)}(P^*) g^{(j)}(P^*) \tag{25}$$

Each $g_k^{(l-j+1)}(P^*) = O(e^{-m})$ by induction assumption (as $l - j + 1 \geq 1$). When $j = 0$, then $g^{(j)}(P^*) = g(P^*) = O(1)$. Therefore the first element of the sum is $g_k^{(l-0+1)}(P^*) g^{(0)}(P^*) = O(e^{-m})$. For all other elements $j > 0$, so the term $g^{(j)}(P^*) = O(e^{-m})$ and therefore each $g_k^{(l-j+1)}(P^*) g^{(j)}(P^*) = O(e^{-2m})$. This proves that $\frac{d^l g_{k+1}(P^*)}{dP^l} = O(e^{-m})$. $\square$

B Calculations for section 4.2

The strategies of the firms are respectively p_1^*, p_2^*, $p_3^*(p_1)$, and $p_4^*(p_1, p_2)$. Let us first consider the problem of player 4, who observes p_1 and p_2 and expects equilibrium behavior from player 3. Therefore player 4 solves

$$\max_{p_4 \geq 0} p_4 \left[1 - p_1 - p_2 - p_3^*(p_1) - p_4\right],$$

which gives us a condition

$$p_4^*(p_1, p_2) = \frac{1}{2}\left[1 - p_1 - p_2 - p_3^*(p_1)\right].$$

While this condition provides a condition for the best-response function $p_4^*(p_1, p_2)$, we have not yet characterized it, as it would require knowing $p_3^*(p_1)$. Player 3 solves a similar problem, but does not observe p_2 and expects p_4 to be $p_4^*(p_1, p_2^*)$, that is

$$\max_{p_3 \geq 0} p_3 \left[1 - p_1 - p_2^* - p_3 - p_4^*(p_1, p_2^*)\right]$$

with the optimality condition

$$p_3^*(p_1) = \frac{1}{2}\left[1 - p_1 - p_2^* - p_4^*(p_1, p_2^*)\right].$$

Again, computing this best-response function explicitly, requires knowing $p_4^*(p_1, p_2)$, but also the equilibrium price of player 2, i.e., p_2^*. To compute the best-response functions explicitly (i.e., independently of each other), we first need to solve the equation system that we get by inserting p_2^* to the optimality condition of player 4. This gives us

$$p_3^*(p_1) = p_4^*(p_1, p_2^*) = \frac{1}{3}\left[1 - p_1 - p_2^*\right] \quad \Rightarrow \quad p_4^*(p_1, p_2) = \frac{1}{3}\left[1 - p_1\right] + \frac{1}{6}p_2^* - \frac{1}{2}p_2.$$

Note the prices p_3 and p_4 we have now characterized are still not the true best-response functions, since they depend on the equilibrium price p_2^*, which is yet to be determined. For this we need to solve the problem of player 2, who expects player 1 to choose equilibrium price p_1^*

$$\max_{p_2 \geq 0} p_2 \left[1 - p_1^* - p_2 - p_3^*(p_1^*) - p_4^*(p_1^*, p_2)\right].$$

Taking the first-order condition and evaluating it at $p_2 = p_2^*$ gives a condition

$$\frac{1}{6}\left[2 - 2p_1^* - 5p_2^*\right] = 0. \tag{26}$$

Finally, player 1 solves a similar problem, taking p_2^* as fixed, i.e.

$$\max_{p_1 \geq 0} p_1 \left[1 - p_1 - p_2^* - p_3^*(p_1) - p_4^*(p_1, p_2^*)\right].$$

Again, taking the first-order condition and evaluating it at $p_1 = p_1^*$ gives

$$\frac{1}{3}\left[1 - 2p_1^* - p_2^*\right] = 0. \tag{27}$$

Solving the equation system equations (26) and (27) gives us $p_1^* = \frac{3}{8}$, $p_2^* = \frac{1}{4}$. Inserting these values to the functions derived above gives us the best-response functions $p_3^*(p_1) = \frac{1}{4} - \frac{1}{3}p_1$ and $p_4^*(p_1, p_2) = \frac{3}{8} - \frac{1}{3}p_1 - \frac{1}{2}p_2$. We can also compute the equilibrium prices $p_3^*(p_1^*) = p_4^*(p_1^*, p_2^*) = \frac{1}{8}$. Therefore equilibrium price of the final good is $P^* = \frac{7}{8}$.

As the example illustrates, finding the equilibrium strategies requires solving a combination of equation systems in parallel with finding the best-response functions. Each additional edge in the network can create a new layer of complexity.

The inverted best-response approach solves this issue as follows. Consider the optimization problem of firm 4. For given (p_1, p_2), it chooses optimal p_4. We can rethink its optimization problem as choosing the final good price $P = p_1 + p_2 + p_3^*(p_1) + p_4$ that it wants to induce. We can rewrite its maximization problem as

$$\max_P [P - p_1 - p_2 - p_3^*(p_1)]D(P).$$

We get the optimality condition

$$D(P) + [P - p_1 - p_2 - p_3^*(p_1)]D'(P) = 0 \iff P - p_1 - p_2 - p_3^*(p_1) = g(P) = 1 - P.$$

This is a necessary condition for optimality, but since the problem is quadratic, it is easy to see that it is also sufficient. This expression gives implicitly the best-response function $p_4^*(p_1, p_2)$. But more directly, the expression on the left-hand side is the optimal p_4 that is consistent with the final good price P and the optimal behavior of firm 4. Let us denote it by $f_4(P) = 1 - P$. The problem for the firm 3 is analogous and gives $f_3(P) = 1 - P$.

Now, consider firm 2. Instead of choosing p_2 it can again consider the choice of the final good price P. Since only firm 4 observes its choice (and thus chooses $p_4 = f_4(P)$ as a response to desired P), the firm 2's problem can be written as

$$\max_P [P - p_1^* - p_3^*(p_1^*) - f_4(P)]D(P),$$

which gives us the optimality condition

$$[1 - f_4'(P)]D(P) + [P - p_1^* - p_3^*(p_1^*) - f_4(P)]D'(P) = 0,$$

or equivalently, $f_2(P) = P - p_1^* - p_3^*(p_1^*) - f_4(P) = 2(1 - P)$. Analogous calculation for firm 1 gives $f_1(P) = 3(1 - P)$. Now, the equilibrium price P^* of the final good must be consistent with individual choices. Therefore we get a condition

$$P^* = \sum_{i=1}^{4} f_i(P^*) = 7(1 - P^*).$$

Solving this equation gives us $P^* = \frac{7}{8}$ and individual prices $p_1^* = 3(1 - P^*) = \frac{3}{8}$, $p_2^* = \frac{2}{8}$, and $p_3^* = p_4^* = \frac{1}{8}$.

Notice that the same calculations could be applied easily for non-linear demand functions, with some $g(P) = -\frac{D(P)}{D(P)}$. This would give us an equilibrium condition

$$P^* = \sum_{i=1}^{4} f_i(P^*) = 4g_1(P^*) + 3g_2(P^*),$$

where $g_1(P) = g(P)$ and $g_2(P) = -g_1'(P)g(P)$. This is again the same pattern that we saw in the previous example, since the number of players is 4 and the number of edges is 3. In the case of linear demand, $g_1(P) = g(P) = 1 - P$ and therefore $g_2(P) = 1 - P$.