Theory of the Firm, Fall 2016

Problem Set 4

Rules: (1) Submission deadline is **October 19th at 16:30** in class or by e-mail (only typed solutions by e-mail). (2) Feel free to consult with your colleagues and any materials, but submit your own solutions. Have fun!

Problem 4.1
Monopolist, who produces with no costs ($c = 0$), knows that consumers get utility $\theta V(q) - T$ when they consume q units and pay T, and 0 when they choose to buy nothing, where $V(q) = q$, and $q \in [0,1]$. Moreover, monopolist also knows that type θ is distributed uniformly in the interval $[0,1]$.

1) What would be the socially optimal quantity as a function of consumer valuation $q(\theta)$?

2) Find the optimal "direct mechanism" $(q(\theta), T(\theta))$.[1]

3) Explain which types and why is not getting their socially optimal quantity?

4) Describe the optimal pricing rule $T(q)$.

Problem 4.2 (Not required)
In the continuum-of-types case we discussed in class, we defined two types of constraints

$$U(\theta) = \theta V(q(\theta)) - T(\theta) \geq 0 \quad \forall \theta, \tag{IR_θ}$$
$$U(\theta) = \theta V(q(\theta)) - T(\theta) \geq \theta V(q(\theta')) - T(\theta') \quad \forall \theta', \forall \theta. \tag{IC_θ}$$

Prove that if the following three conditions hold, then (IR_θ) and (IC_θ) are satisfied:

1) $(IR_{\underline{\theta}})$

2) "Monotonicity": $\theta' > \theta \Rightarrow q(\theta') \geq q(\theta)$.

3) "Envelope condition": $U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} V(q(t))\,dt$.

[1] Hint: we showed in class that the optimal mechanism is defined by the first-order conditions

$$\left[\theta - \frac{1 - F(\theta)}{f(\theta)}\right] V'(q(\theta)) = c$$

and the envelope condition

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} V(q(t))\,dt,$$

where at the optimum $U(\underline{\theta}) = 0$. Feel free to use these conditions.

Hint: we proved very similar result for two-type case in class.
Note: for continuum-type case we proved the other direction: (IR_θ) and $(IC_\theta) \Rightarrow$ (1)-(3).

Problem 4.3
Suppose that seller has a painting that she wants to sell and there are n bidders with independent private values, where bidder i's valuation θ_i is distributed uniformly in $[0,1]$.
Suppose that the seller sells the painting at a sealed-bid second price auction (SPA) without reserve price, where the bidders simultaneously submit bids $\mathbf{b} = (b_1, \ldots, b_n)$, highest bid wins, and the winner pays second-highest bid to the seller.
[If the calculations seem complicated, feel free to assume that $n = 2$.]

1) Describe SPA as a game, i.e. describe: (1) players, (2) strategies, (3) payoffs.

2) Prove that everyone bidding their true valuation is an equilibrium.

3) Verify that SPA is an efficient mechanism (i.e. welfare-maximizing). For this

   (a) Argue that it always allocates the object to the bidder with highest valuation.

   (b) Compute the expected transfers, i.e. the expected amount that a bidder with value θ_i expects to pay.

   (c) Verify that the expected transfer expression is equal to $T_i(\theta_i)$ we derived in class for the efficient mechanism.

4) Is this SPA profit-maximizing? If not, explain intuitively, how we can increase the profits.

Problem 4.4
Exercise 5.1* (page 211)[2]

Problem 4.5
Consider duopoly facing inverse demand function $p = P(Q) = a - bQ$ (or demand function $Q = D(\underline{p}) = \frac{1}{b}(a - \underline{p})$), where $Q = q_1 + q_1$ and $\underline{p} = \min\{p_1, p_2\}$. Suppose marginal cost is c for both firms.

1) Suppose that firms form a cartel where they both agree to sell at the same price. Find equilibrium prices, quantities, and profits.

2) Suppose that firms compete in prices. Find Bertrand equilibrium prices, quantities, and profits.

3) Suppose firms agreed to set cartel prices, but one firm chooses to cheat. What is the optimal price and what are the profits.

4) Suppose now instead that firms compete in quantities. Find Cournot equilibrium quantities, price, and profits.

5) Suppose that firms form a cartel where firms produce equal quantities. Find optimal quantities, price, and profits.

6) Suppose that firms agreed to produce cartel quantities, but one firm chooses to cheat. What is its optimal quantity and what are the profits.

[2] All references are to Tirole's book "The Theory of Industrial Organization".