Theory of the Firm, Fall 2016

Problem Set 5

Rules: (1) Submission deadline is **November 2nd at 16:30** in class or by e-mail (only typed solutions by e-mail). (2) Feel free to consult with your colleagues and any materials, but submit your own solutions. Have fun!

Problem 5.1

Consider duopoly facing inverse demand function $p = P(Q) = a - bQ$ (or demand function $Q = D(\underline{p}) = \frac{1}{b}(a - \underline{p})$), where $Q = q_1 + q_1$ and $\underline{p} = \min\{p_1, p_2\}$. Suppose marginal cost is c for both firms.

1) Suppose that firms compete with prices (Bertrand game) infinitely many times and have common discount factor δ. Find the set of discount factors for which full collusion is possible.

2) Suppose that firms compete with quantities (Cournot game) infinitely many times and have common discount factor δ. Find the set of discount factors for which full collusion is possible.

Problem 5.2

Exercise 6.6* (page 251)

Problem 5.3

Exercise 6.4* (page 250)

Problem 5.4

Exercise 7.2** (page 282)[1]

Problem 5.5

Consider the linear city model with linear costs. That is, consumers are distributed uniformly in the interval $[0, 1]$ and utility from buying from firm i is $\bar{s} - td_i - p_i$, where d_i is distance to firm i's location. There are two stores. Store 1 is located at a and Store 2 is located at $1 - b \geq a$. Both stores have constant marginal cost c. Denote $\Delta = 1 - a - b$. Suppose that the assumptions we made in class hold, i.e. all consumers buy from one of the stores and both stores sell positive quantity.

1) Derive demand functions.

2) Suppose that the locations (a, b) are fixed and stores choose prices (p_1, p_2) simultaneously. Suppose that the equilibrium is such that both firms sell positive quantities a get positive profits. Find the equilibrium prices.

3) Consider the case of maximal differentiation. What are the prices? What are the profits?

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[1]All references are to Tirole's book "The Theory of Industrial Organization".

4) Consider the case of no differentiation. Is the equilibrium you found earlier consistent with Bertrand paradox? Why?