Theory of the Firm, Fall 2016

Problem Set 7

Rules: (1) Submission deadline is **November 14th at 9:00** in class or by e-mail (only typed solutions by e-mail). (2) Feel free to consult with your colleagues and any materials, but submit your own solutions.

Have fun!

Problem 7.1 **

Consider the basic model of sequential search: agent gets one random draw x_t each period, where $\{x_t\}$ are i.i.d. draws from distribution F. Suppose search cost $c = 0$, but waiting is costly because of discounting. In particular, getting prize x_t in period t is worth $\delta^t x_t$. Assume that the decision maker is using a reservation value stopping rule, with reservation value y.

1) Let $V(y)$ denote the expected discounted utility that the decision maker expects from using reservation value y^*. Show that $y^* = \delta \frac{\int_{y^*}^{\infty} x dF(x)}{1-\delta F(y^*)} = \delta V(y^*)$.

2) Suppose the support of distribution F is $[0, \overline{x}]$. What are the limits of y^* when $\delta \to 0$ and $\delta \to 1$. Interpret.

3) Find y^*, when $\delta = \frac{1}{2}$ and each x_t is distributed uniformly on $[0,1]$.

Problem 7.2 *

Suppose you are offered lottery tickets, $L_1, L_2, L_3, \ldots$ (one for each natural number), where L_n costs p_n and pays out n with probability $\frac{1}{2}$ and 0 with probability $\frac{1}{2}$. You can buy and play lotteries sequentially, but there are two constraints: 1. you can only play each lottery once, 2. even if you get multiple winning lottery tickets, you can choose only one prize.

1) Calculate expected value of buying (only) lottery L_n.

2) Suppose price of L_n is $\frac{n^2}{16}$ for each $n \in \mathbb{N}$. Which lottery should you play first? [Hint: treat n as continuous variable.]

3) Assuming still that the price of L_n is $\frac{n^2}{16}$, describe the optimal way to play these lotteries.

Problem 7.3 **

Consumer wants to buy an object for which she is willing to pay at most $v = 100$ and that has list price $P = 100$. In particular, she gets utility $v - p$ when she buys the object at price p and 0 when she does not buy the object.

Suppose that the are 100 stores in the area that sell this product, but she does not know the exact prices. She only knows that for each $n = 1..100$, the store n makes (independent) random discounts of $d_n \in [0, 1]$ from the list price, where d_n is distributed uniformly[1] in $[0, n/100]$. That is, the discounted price at store n would be $p_n = (1 - d_n)100$.
She can check the prices sequentially and decide after each store visit which of the offers to take (and potentially continuing searching).

1) Suppose that visiting store n costs $c_n = c = const$, e.g. the distance to each store is equal.

   (a) Calculate expected value of checking out (only) store n.

   (b) For which values $c > 0$ she will not check any stores?

   (c) If she checks some stores, which should be the first one? Explain intuitively, why.

   (d) Under which conditions she should go to more than one store?

   (e) Under which conditions she would go to all the stores?

2) Suppose that the cost of visiting store n costs $c_n = \frac{n}{2}$. Again:

   (a) Calculate expected value of checking out (only) store n.

   (b) If she checks some stores, which should be the first one? Explain intuitively, why.

   (c) Under which conditions she should go to more than one store?

   (d) Under which conditions she would go to all the stores?

Problem 7.4 *
Give real-life examples for each of the following:

1) Costly search.

2) Sales as price discrimination tool.

In each case: (1) be specific and name specific product or even firms, (2) explain how the example fits, (3) but still try to limit your answers to 1–3 sentences for each part.

[1] For example, Store 3 has discount policy such that it has discounts between 0–3%, whereas store 97 has discounts between 0–97%.